MACHINE DESIGN.

PART I.

KINEMATICS OF MACHINERY.

BY

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PREFACE.

IN these notes an attempt is made, first, to give, as clearly and concisely as possible, the principles of mechanical motion in such a manner that their application can readily be made to any mechanism for determining the motion of any of its parts; then to show the methods of dealing with such problems as the designer meets daily. Long and tedious discussions have been avoided as far as possible, it is hoped, fully.

Subjects such as toothed gearing and couplings are taken up only to the extent of the forms that are in the most common use. But with these subjects, as well as all others, references to what are believed to be the best works in their lines are given frequently.

All available works on the subject have been freely consulted, but in no case has any matter which has not become common property by its frequent publication been used without the consent of its author.

The exceedingly clear and concise work of Prof. Albert W. Smith, of Stanford University, entitled "Machine Design," has been of most valuable assistance throughout. This work includes both kinematics and mechanics. To Prof. Smith, especially, the writer would acknowledge his obligations and express his thanks.

FORREST R. JONES.

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KINEMATICS OF MACHINERY.

CHAPTER I.

GENERAL PRINCIPLES AND DEFINITIONS.

MOTION OF A BODY.

1. When a body moves, there must always be another body with regard to which the motion occurs. Sometimes the statement that the movement takes place is all that is necessary to define it, the conditions being such that the reference body is clearly implied; but at other times a more specific statement is required. A simple example will illustrate: When a boat is running through the water at the rate of 12 miles an hour against a current of 3 miles an hour, the motion is clearly 12 miles an hour relatively to the water; but when referred to the land it is $12 - 3 = 9$ miles an hour.

The wheel of a locomotive furnishes another example: As the locomotive passes along the track, the wheel simply rotates with regard to the frame of the engine; but when referred to the track, the motion is a combined one of rotation and translation.

An examination of the motion of the piston shows a somewhat similar case: Relatively to the locomotive, the motion is reciprocating, its path being back and forth from end to end of the cylinder; but the motion is always forward with regard to the track when the locomotive moves forward, and *vice versa.*

A body entirely free to move may have motion in any direction according to the influences brought to bear upon it. In order for the motion to be a useful one, it must be constrained to such an extent that it will fulfil its required functions.

2. The three principal forms of constrained motion are as follows:

1st. **Plane motion.**—If a body having a plane surface on one side is placed against the plane surface of another body and moved so that the surfaces always remain in contact, the first has a plane motion relatively to the second, whatever the nature of the motion otherwise.

Thus, if a book is placed with one side on a table and slipped about without lifting its side from the table, it has a plane motion, due allowance being made for irregularities of the surfaces.

If a body does not have a plane side, it is possible to give it a plane motion by attaching three points to it so that they will rest and move on the reference plane, or by attaching it rigidly to a rotating shaft, or by the use of other suitable devices.

In general, *a body has plane motion when it moves so that a plane can be passed through it cutting a section which will coincide with the cutting plane throughout the motion.*

Rectilinear motion occurs when every point in a body moves in a straight line.

Rotary motion takes place when every point in a body moves in a circle.

2d. **Helical motion.**—When an ordinary screw is turned about its axis so as to pass through a nut with which it engages, it has a helical motion relatively to the nut. Every point in the screw, except those in its axis, has a combined motion of rotation and translation, the ratio of the magnitudes of these two elementary motions being constant for any point in or attached to the screw.

3d. **Spherical motion.**—If a rigid body is attached to another by a "ball-and-socket" joint which will allow it to move in any direction about the centre of the ball and socket, then every point in the body has a spherical motion when the body is moved in more than one direction about the joint. (Movement about the centre in one direction only would give the body a rotary motion.)

Spherical motion may be defined as the motion of a body moving so that every point in it remains at a constant distance from a centre of motion, but does not remain in a plane.

3. Relative motion.—Two bodies are said to have the same motion when they can be rigidly connected together during the

motion without changing it in any way. From this it is evident that—

1st. **If any two bodies have the same motion relatively to any other, they have no relative motion, and vice versa.**

2d. **The relative motion of two bodies is not affected by any motion common to both.**

Any motion of a body can be resolved into two simple ones—one of translation and one of rotation; therefore, if two bodies

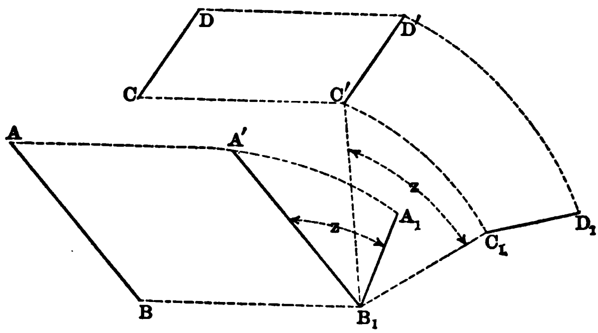

FIG. 1.

have the same motion, both can have their motions resolved into equal translations and equal angular rotations about the same axis.

Illustration.—In Fig. 1, let AB and CD be the initial positions of two bodies having the same motion, the final positions being A_1B_1 and C_1D_1. The change of position of AB can be accomplished by two motions, the one a translation from AB to $A'B_1$, and the other a rotation about an axis through B_1, passing through an angle z into the position A_1B_1.

The motion of CD can be resolved into an equal and similar translation from CD to $C''D'$, and an equal rotation z, about the same axis through B_1, into the position C_1D_1.

(It should be remembered that the bodies may be considered as being rigidly fastened together.)

When, starting from given positions, the (different) motions of two bodies are known with regard to the same standard, the motion of one relatively to the other may be found by giving both the same motion of such a nature that one will return to its original

position; then the difference of the initial and final positions of the other will be its motion relatively to the first.

Illustration.—In Fig. 2, let A and B be the initial positions of two bodies, A_1 and B_1 being their final positions relatively to the same standard (in this case the paper). Their relative motion while thus changing their positions, is found by giving both a motion the reverse of that which B has passed through. By doing this, B is brought back to its initial position, while A is rotated from A_1 to A' and then translated to A_2. The difference between the posi-

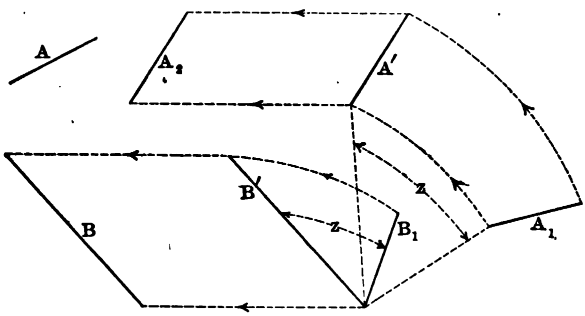

FIG. 2.

tions A and A_2 is the motion of A relatively to B when the two bodies move from A and B to A_1 and B_1. By this method any number of equal and similar motions can be added to or taken from the motions of two bodies, dealing with each motion successively.

If both bodies had been given a reverse motion equal to that of A (instead of that of B), thus bringing A back to its original position, the motion of B relatively to A would have been shown to be equal and *opposite* to that of A relatively to B. Hence it may be stated that **if any two bodies, A and B, move relatively to each other, the motion of B relatively to A is the same as that of A relatively to B, but of opposite sense.** This is true under all conditions.

Plane Motion.

4. The motion of a point relatively to a line is determined by its motion relatively to two points in the line. This assumes that the position of the point at any instant is located by its distances from

the two points of the line, there being no other way to locate two points relatively to each other when no auxiliary plane of reference is given.

The motion of a line relatively to another line in the same plane is determined by the motion of any two points of the one relatively to two points of the other. This depends on the fact that the positions of any two points in a line at any instant determine the position of the line at that instant. It is evident that if the motion of a line (or two points) in a plane body moving in its own plane or parallel to it is known, the motion of the body is determined; for, the body being rigid, all points in it have the same motion; also, any point in the body can be located at any instant by its distance from the points in the line.

Although, strictly speaking, a point is not positively located in a plane when its distances from two points in the line of the plane are known, since it may occupy either of two positions on opposite sides of the line, there are, in the constrained motions of the mechanisms to be considered later, always conditions sufficient to locate the point definitely. The same is true with regard to bodies.

5. Instantaneous motion and instantaneous axis.—When a body changes its position, its motion at any instant is called its

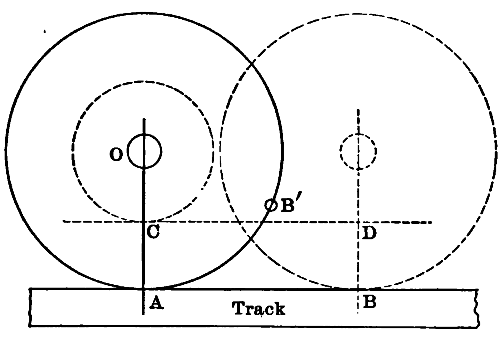

FIG. 3.

instantaneous or *virtual* motion for that instant. (The words "instantaneous" and "virtual" are interchangeable in all cases.)

Take, for example, a locomotive wheel rolling along the track as the engine moves forward. For an instant there is contact between the wheel and track along a line at A, Fig. 3, perpendicular to

the plane of the wheel and paper. Since there is contact along a line at A, which thus becomes an element of both wheel and track, the wheel rotates about A relatively to the track *for an instant*. This is the instantaneous motion of the wheel relatively to the track. The line of contact at A is the instantaneous or virtual axis.

When the engine has moved forward a distance AB, the point B' on the wheel comes in contact with the track at B, and at that instant the wheel rotates about B as an instantaneous axis. During the motion of the wheel from the first to the second position, the instantaneous axis occupies, at each instant, a different position on AB. AB, therefore, contains all the instantaneous axes for the given motion, and, for this reason, is called the axode of the wheel relatively to the track for the given motion.

If, instead of the entire wheel, a section made by a plane perpendicular to its axis is considered, the point at which the instantaneous axis pierces it is called the **instantaneous centre** or **centro**, and the locus of such points is the **centrode**.

Now suppose that the wheel, instead of simply rolling along the track, slips so that it makes two revolutions while passing, at a uniform speed, over a length of track equal to its circumference, the rate of the rotation also being uniform. Then the instantaneous axis for any instant evidently does not lie at the contact line. The motion of the wheel, since it turns twice as rapidly as when having a purely rolling motion on the track, is as if it had an additional tread of half the diameter of the first, rolling upon a rail midway between A and O without slipping between this auxiliary tread and its rail. Hence the centro would be midway between the real rail and the axis of the wheel. For any other rate of slipping, as long as the rotation is more rapid than required for a purely rolling motion on the track AB, the instantaneous axis will be at some point between A and O, on a line joining the two points. If, on the contrary, the wheel is prevented from rotating freely by a brake applied to it so as to cause partial slipping on the track, the centro will still lie on a line passing through A and O, but on the side of A opposite O. If the brake completely locks the wheel, the instantaneous axis may be considered as at an infinite distance from O, but still on the line OA.

The centro for the motion of the track relatively to the wheel,

is the same at any instant as that of the wheel relatively to the track. This assumes the wheel as standing still and the track moving. The centrode is the circle bounding the plane section cut from the wheel by the plane of the paper. When slipping occurs, making the revolutions twice as rapid as for rolling contact, the centrode becomes a circle having half the diameter of the wheel tread.

6. Location of centros in a single body.—When a rigid body revolves about any axis, the direction of motion of any point in the body is perpendicular to the line drawn from the point and normal to the axis of rotation. Conversely, the axis of rotation will intersect a line drawn normal to the motion of any point in the body and lying in the plane of the motion of the point. These statements are applicable to motion about both permanent and instantaneous axes.

If the directions of motion of any two points lying and moving in the same plane are known at any instant, the centro can be found by drawing through each point a line perpendicular to the direction of its motion. Their intersection gives the centro. Thus, in Fig. 4, *A* and *B* are any two points lying and moving in the

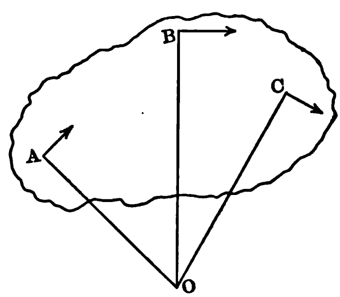

FIG. 4.

same plane at any instant, in the direction indicated by the arrows. By drawing *AO* perpendicular to the direction of the motion of *A*, and *BO* perpendicular to that of *B*, their intersection at *O* is the instantaneous centre. If the lines through *A* and *B*, perpendicular to their directions of motion, coincide, there can be no intersection, and the centro is not determined. When the lines are parallel but

not coincident, the instantaneous centre is at an infinite distance, the motion of the body being translation only.

Having found the centro, the direction of motion of any point in the body, as C, is perpendicular to the line OC.

If the points A and B are the projections of points moving in different planes parallel to the paper, then O is the projection of the instantaneous axis, which, of course, is perpendicular to the paper.

7. Notation.—The centro of two bodies, as A and B, moving relatively to each other will be indicated by *ab* or *ba*, both meaning exactly the same. Either may be read "the centro" or "the virtual (or instantaneous) centre" of A relatively to B, or "the centro" or "the virtual (or instantaneous) centre" of B relatively to A. The order of the letters is of no consequence. It should be remembered that the centro of A relatively to B is the same as that of B relatively to A.

The centro of A and C is *ac* or *ca*; of B and C, *bc* or *cb*.

8. Positions of centros in three bodies.—In Fig. 5, A, B, and C are three bodies moving in the same plane. It is desired to show

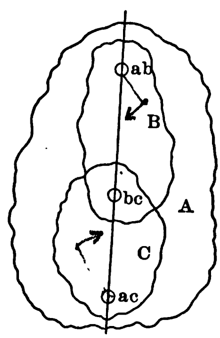

FIG. 5.

that the three centros lie in the same straight line. Since, if any number of bodies have the same motion added to or subtracted from them, their relative motion remains the same, two proofs will be given, one considering B as stationary, and the other for A stationary.

1st proof.—B is assumed as stationary and *ab* and *ac* are given in location; they may be either permanent centres or centros. Since

ab is stationary, the only motion *A* can have is that of rotation about *ab*. At the same time that *A* is rotating about *ab*, *C* must rotate about *ac* relatively to *A*, this being the only motion that *C* can have. As *C* rotates about *ac*, every point in *C*, *except those on a straight line passing through ac and ab*, changes its distance from *ab*, and therefore cannot be stationary, as *bc* must be since it is a point in the stationary body *B*. Therefore *bc* must lie on the straight line passing through *ab* and *ac*.

2d proof.—*A* is taken as stationary while *ab* and *ac* are known in location as before, and are also stationary. Every point in *B* moves perpendicular to the radial line passing through it and *ab*; also, every point in *C* moves perpendicular to the radial line passing through it and *ac*. Therefore, for any point in *B* and one in *C* to have the same motion, both points must move at right angles to both radial lines. This requires the lines to be either parallel or coincident. They cannot be parallel, for they must pass through the same point *bc*, but must coincide. Since they coincide, both must pass through *ab* and *ac*. Therefore the centros *bc*, *ac*, and *ab* lie on the same straight line.

In accordance with the above it can be stated that **if *A*, *B*, and *C* are any three bodies moving relatively to each other, their centros *ab*, *ac*, and *bc* lie on the same straight line.**

9. A kinematic chain is a combination of rigid bodies so connected that the motion of each is either completely constrained and depends on the motions and positions of the others, or, as in cases

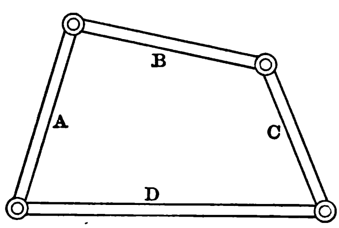

FIG. 6.

where a part is actuated by a spring, its own weight, or a corresponding device, the motion of the part must occur during certain motions of the others or while they occupy definite positions. Figs. 6 and 7 are examples of kinematic chains. The links *A*, *B*, *C*, and

D are so connected that they can move relatively to each other, each occupying a definite position, at any instant, determined by the positions of the others.

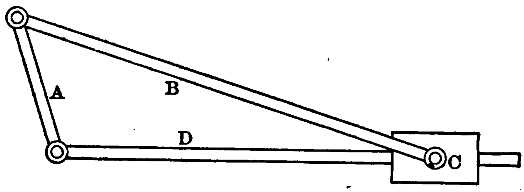

FIG. 7.

In Fig. 6 each link rotates relatively to the ones to which it is connected. In Fig. 7 the links *C* and *D* have a sliding or parallel

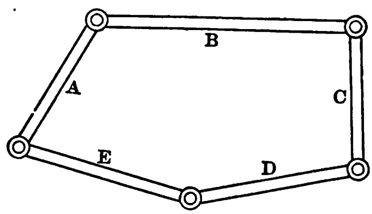

FIG. 8.

motion relatively to each other. Fig. 8 is not a kinematic chain, for the motion of no link is completely constrained, and the move-

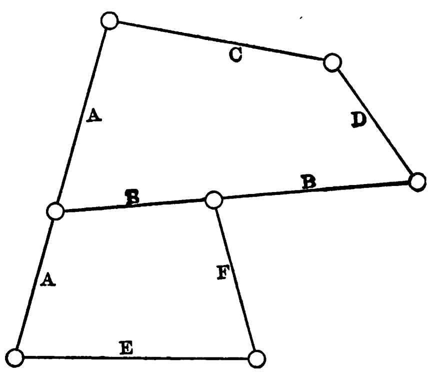

FIG. 9.

ment of one link does not give any definite motion to any other. (Fig. 8 would be called a kinematic chain by some writers.)

The parts of two connecting links forming the articulation between them are called a *pair of elements.* If the relative motion of the elements is rotation, they form a *turning pair;* when the relative motion is translation or sliding, they are a *sliding pair.* In Fig. 7 the elements joining A and B are a turning pair; those joining C and D, a sliding pair.

The links of a kinematic chain are conventionally represented by straight lines connecting two elements belonging to different pairs of elements. They can be of any form, however, provided they do not interfere with the motion of the others by striking against them.

MECHANISMS.

Determination of Centros.

10. When one link of a kinematic chain is assumed to be stationary, or when it is taken as the body to which all motions of the other parts are referred, the combination is generally called a mechanism or machine.

11. Simple lever-crank chain of turning pairs.—Fig. 10. Since the centros of adjacent links are at their articulations, *ad, ab, bc,*

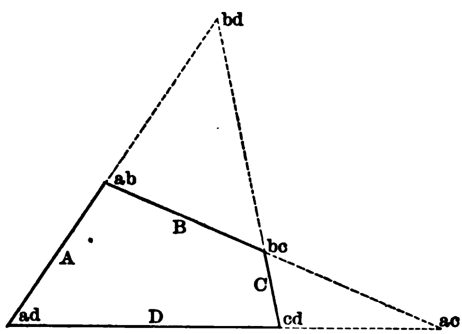

Fig. 10.

and *cd* are readily determined. To find *bd,* suppose D to be stationary; then *ab,* which is a point in B, moves at right angles to a line through *ab* and *ad*; therefore the centro *bd* must lie on A. Also, *bc* is another point in B, and moves at right angles to a line through *cd* and *bc,* which limits the position of *bd* to some point on C. Since *bd* lies on both A and C, it must be at their intersection,

as indicated in the figure. The centro *ac* can be located in a similar manner by considering *A* at rest.

Again *bd*, can be determined by making use of the fact that *ad*, *ab*, and *bd* must lie on the same straight line (§ 8), while *cd*, *bc*, and *bd* have the same property; *bd* must, therefore, lie at the intersection of these lines. The location of *ac* can be determined in the same manner.

12. Simple slider-crank chain, one sliding and three turning pairs.—In Fig. 11, *ab*, *ad*, and *bc* are the joins of the links indicated by *A*, *B* and *C*. To find *bd*, assume *D* to be at rest; then *ab*, which is a point in *B*, moves about *ad* at right angles to *A*; therefore *bd* must lie on *A*, for it is also a point in *B* and must move about the same centro. Also, *bc*, which is a point in *B*, moves in a direction parallel to *D*; therefore *bd* must lie on a line perpendicular to *D* at *bc*. The intersection of this line with *A* prolonged is, therefore, the position of *bd*.

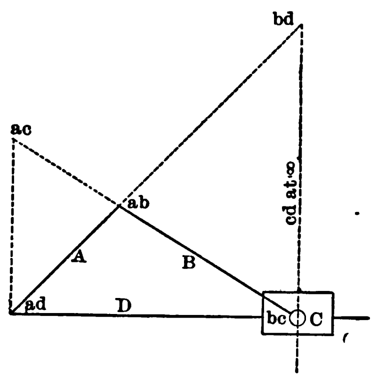

FIG. 11.

Now assume *C* at rest in order to locate *ac*; then *ab*, which is a point in *A*, moves about *bc* at right angles to *B*; therefore *ac* must lie on *B*. Also, *ad*, which is a point in *A*, moves in a line parallel to *D*; therefore *ac* must lie on a perpendicular to *D* at *ad*. The intersection of these lines determines *ac*.

When *D* is at rest, *C* moves parallel to it. The centro *cd* is, therefore, at an infinite distance from these links and on lines perpendicular to *D*. The same can be shown by assuming *C* at rest.

13. Simple chain, two sliding and two turning pairs.—Fig. 12. The link *A* is connected to *B* and *D* by turning pairs; *B* slides along the vertical part of *C*, while the horizontal part of *C* slides through the right-hand end of *D*.

The articulations of *A* with *B* and *D* are at *ab* and *ad*. To find *ac*, assume *C* at rest; then *ad*, which is a point in *A*, moves in a

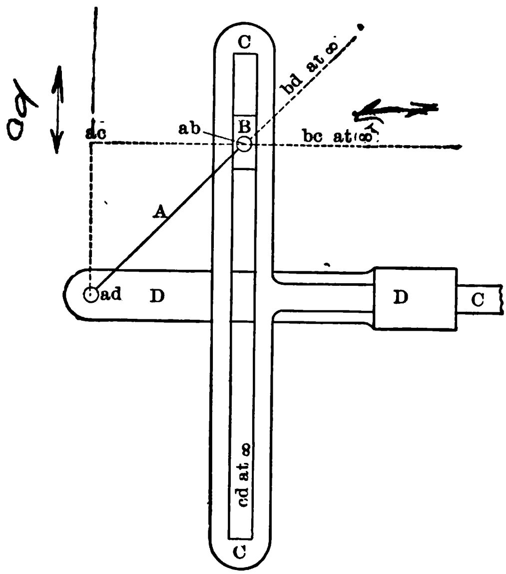

FIG. 12.

direction parallel to *D*; therefore *ac* must lie on a perpendicular to *D* at *ad*. Also *ab*, which is a point in *A*, moves at right angles to *D*; therefore *ac* must lie on a parallel to *D* passing through *ab*. The intersection of the two lines upon which *ac* must lie determines the location of *ac*.

When *C* is at rest, *D* moves parallel to the horizontal part of *C*. The centro *cd* must, therefore, be at an infinite distance on lines perpendicular to *D* and the horizontal part of *C*. The same is true for *B*, *C*, and *bc*.

To determine *bd*, assume *D* at rest; then *ab*, which is a point in *B*, moves about *ad* as a centre; therefore *bd* must lie on *A*. *B* does not rotate relatively to *D*, its motion being one of transla-

tion only. This shows that *bd* must lie at an infinite distance from the mechanism, which locates *bd* on *A* at infinity.

14. Compound chain.—In Fig. 13, *ab, ae, ad, cd, bc*, and *ef* are the articulations of the given links. *ac* and *be* are located as in § 12.

By making use of the fact that the centros of three bodies lie in the same straight line (§ 8), *de* and *ce* can be located. To do this, first take the links *A, C*, and *E*; then *ac, ae*, and *ce* must lie on the same line. Two of these points, *ac* and *ae*, have been deter-

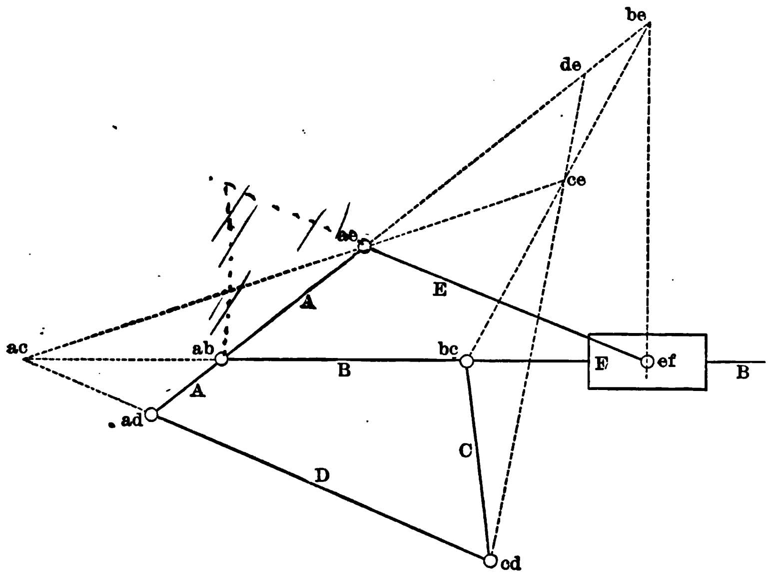

Fig. 13.

mined, and therefore one of the lines containing *ce* is determined, since it passes through *ac* and *ae*. Next take *B, C*, and *E*. *bc, be*, and *ce* must lie on the same line; *bc* and *be* are known in position, therefore another line containing *ce* is determined, for it passes through *bc* and *be*. The intersection of the line through *bc* and *be* with that through *ac* and *ae* determines *ce*. By taking the links *A, D, E*, and *C, D, E, ed* can be located in the same manner.

All the remaining centros can be determined by the method just applied.

Relative Linear Velocities.

15. When a rigid body rotates or oscillates about a permanent centre, the linear velocity of any point in the body is proportional to its radial distance from the centre. The same is true for virtual linear velocities about a centro.

The distance of a point from the centro is called its **virtual** (or instantaneous) **radius.** When the virtual radius is infinitely long, the linear velocity of every point is the same.

V–, will be used to represent the linear velocity of a point or a body in which all points move at the same rate. Thus, V–.H is read "the linear velocity of H."

As a means of indicating a link that is at rest, short lines are drawn at an acute angle on one side of it.

16. Linear velocity about a permanent centre.—*1st method.*—In Fig. 14 let H and N be any two points in the link A, the velocity

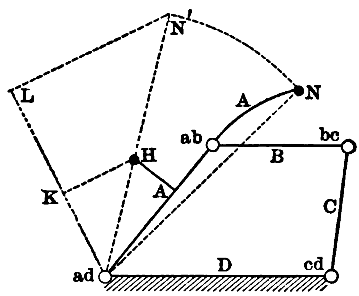

FIG. 14.

of H being given and that of N required about the permanent centre *ad*.

Lay off, in any direction from H, a distance $HK = V\text{–}.H$ according to any convenient units and scale of drawing. Draw $(ad)KL$, take $(ad)N' = (ad)N$, and draw $N'L$ parallel to HK. By the similar triangles $KH(ad)$ and $LN'(ad)$,

$$\frac{N'L}{HK} = \frac{N'(ad)}{H(ad)} = \frac{N(ad)}{H(ad)} = \frac{V\text{–}.N}{V\text{–}.H}.$$

But $HK = V\text{–}.H$; therefore

$$\frac{N'L}{V\text{–}.H} = \frac{V\text{–}.N}{V\text{–}.H}; \quad \text{or} \quad N'L = V\text{–}.N.$$

2d method.—In Fig. 15, H and N are any two points in the link A, as before. Draw HN, $H(ad)$, and $N(ad)$. Take $HK = V\!-\!.H$, and draw KL parallel to HN, intersecting $N(ad)$ at L. By the similar triangles $HN(ad)$ and $KL(ad)$,

$$\frac{NL}{HK} = \frac{N(ad)}{H(ad)} = \frac{V\!-\!.N}{V\!-\!.H}.$$

But $HK = V\!-\!.H$; therefore

$$\frac{NL}{V\!-\!.H} = \frac{V\!-\!.N}{V\!-\!.H}; \quad \text{or} \quad NL = V\!-\!.N.$$

If the centre of rotation and the given points H and N lie on or near the same straight line, it becomes necessary to take an auxiliary point whose velocity is first found, and from this velocity that of the given point. Thus, in Fig. 15 the points h and n are on the link C, the velocity of h being known and that of n required.

The points h, n, and cd are so nearly in a straight line that the intersection of $n(cd)$ with a line drawn through k parallel to one joining h and n could not be accurately located. The point p is, therefore, taken in any convenient position, and, by the method just given, its velocity is found to be pl. Then that of n is readily obtained, and is ns.

The second method represented in Fig. 15, with and without the auxiliary point, is useful when the centre of rotation is inac-

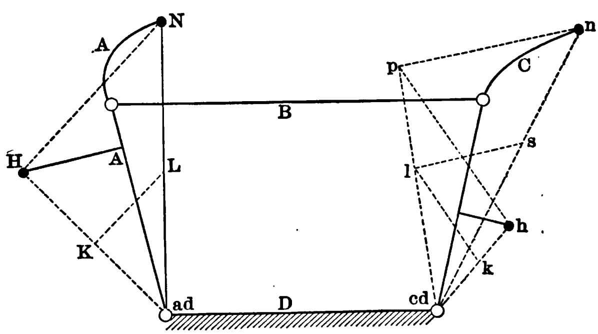

FIG. 15

cessible, as is frequently the case for centros. This usefulness depends on the fact that the lines passing through the given points,

and the auxiliary one when used, need only be drawn toward the centre of rotation, there being no necessity of their extending to it. (See § 18.)

17. Linear velocity about an accessible centro.—In Fig. 16, *H* and *N* are any two points in the link *B*. The velocity of *H* being given, that of *N* is required.

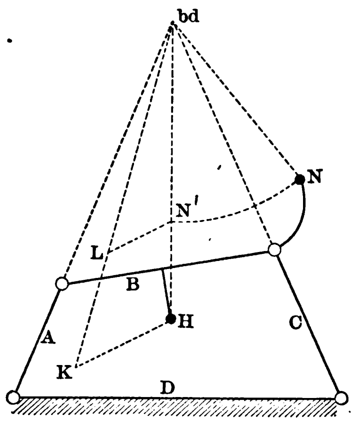

FIG. 16.

The solution is the same as that of the first method in § 16, the centro of revolution in this case being *bd*. *HK* is taken, in any direction, to represent the velocity of *H*. *N'*(*bd*) is taken equal to *N*(*bd*), and *N'L* is drawn parallel to *HK*, which gives *N'L* as the velocity of *N*.

18. Linear velocity about an inaccessible centro.—Fig. 17. *H* and *N* are points in the link *B*. The velocity of *H* being given, that of *N* is required. The centro of rotation for *B*, which is *bd*, lies at the intersection of the line through *ab* and *ad*, with the line through *bc* perpendicular to *D*. Since the directions of these two lines passing through *bd* are known, lines can be drawn through *H* and *N* in the direction of *bd*, by the geometrical method of drawing a line in the direction of an inaccessible point when the directions of two lines passing through it are known. *HK* and *LN* are lines so drawn. The problem can now be completed by the second method of § 16. The solution is as follows: Join *H* and *N*, draw

HK and *NL* toward *bd*, take *HK* as the velocity of *H*, draw *KL* parallel to *HN*; then *NL* represents the velocity of *N*.

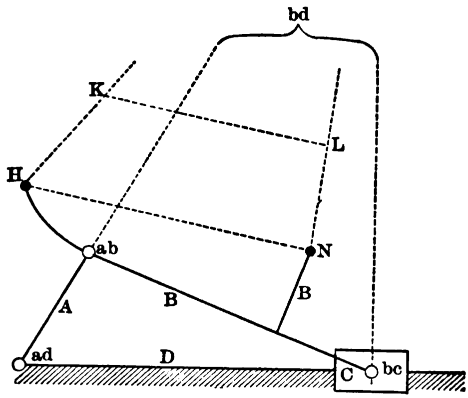

Fig. 17.

19. Linear velocity of points in different links.—In Fig. 18, *H* and *N* are any two points in the links *A* and *B* respectively. The velocity of *H* is known, and that of *N* required.

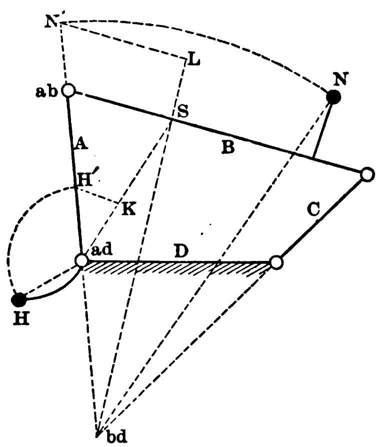

Fig. 18.

The solution requires the use of an auxiliary point which must be common to both links, and is, therefore, their articulation, which, in this case, is *ab*. The first method of § 16 is applicable.

Take $H'(ad) = H(ad)$ and draw $H'K$ parallel to B, making $H'K$ equal the velocity of H; draw $(ad)K$ cutting the link B at S; then $(ab)S$ is the velocity of ab, which is a point in B. The centro of D is bd, and the velocity of ab about bd is $(ab)S$. Now take $N'(bd) = N(bd)$; draw $N'L$ parallel to B, and $(bd)S$ cutting $N'L$ at L; then $N'L$ is the velocity of N.

The similar triangles used for finding the velocity of ab from that of H are $H'K(ad)$ and $(ab)S(ad)$; those for finding that of N from the velocity of ab are $(ab)S(bd)$ and $N'L(bd)$.

Angular Velocities.

20. The angular velocity of a body rotating or oscillating about a centre or centro is the same for every point in the body. If, in two bodies, there are two points having the same linear velocity, but different radial distances from their centres of rotation, their angular velocities are inversely as their radii. Thus, if P and Q are two points having radial distances of 2 feet and 6 feet respectively, the linear velocity of both being 10 feet per second, P will make three revolutions while Q is making one, which gives the angular velocity of P three times that of Q, or inversely as the radial distances of the points having the same linear velocity in each.

In engineering practice, angular velocities are generally measured by revolutions per minute. Other units of angular and time measure may be used, as revolutions per second or hour, degrees or radians per minute or other time unit, etc.

21. Relative angular velocities of links.—In Fig. 19 suppose that the angular velocity of A relatively to D is given, and that of B relatively to D is required.

$V°$ is used to indicate the angular velocity of a body. Thus, $V°A$ is read "the angular velocity of A."

For convenience the link D will be assumed at rest. The point common to both A and B is ab. In either body it has the same linear velocity. As a point in A its radial distance is $(ab)(ad)$, and as a point in B its radial distance is $(ab)(bd)$, from which

$$\frac{V°A}{V°B} = \frac{(ab)(bd)}{(ab)(ad)}.$$

Therefore

$$V^\circ B = \frac{(ab)(ad)}{(ab)(bd)} \cdot V^\circ A.$$

The graphical solution is shown in Fig. 19. It is as follows: Join ab and cd; take $(ab)G$ on any convenient scale, as $V^\circ A$; draw

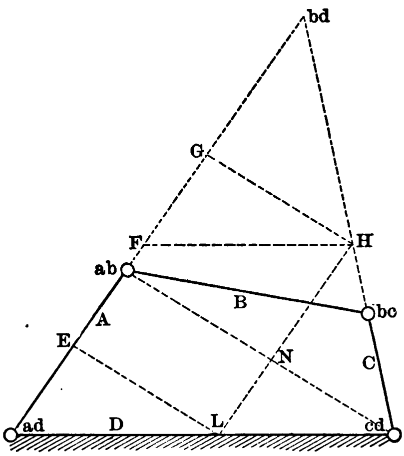

FIG. 19.

GH parallel to $(ab)(cd)$; draw HL parallel to $(bd)(ad)$ intersecting $(ab)(cd)$ at N; then, by similar triangles and since $HN = G(ab)$ by construction,

$$\frac{HN}{NL} = \frac{(ab)(bd)}{(ab)(ad)} = \frac{V^\circ A}{V^\circ B}.$$

And since $HN = V^\circ A$, therefore $NL = V^\circ B$ on the same scale of drawing.

If $V^\circ B$ is given and $V^\circ A$ required, both relatively to D, take $(ab)E = V^\circ B$; draw EL parallel to $(ab)(cd)$, and HL parallel to $(bd)(ad)$; then $NH = V^\circ A$. This construction and proof are similar to that when $V^\circ A$ is given.

If $V^\circ B$ is given and $V^\circ D$ required for the opposite link D, one of the intermediate links, as A, can be assumed to be stationary. The point bd is a point in both B and D; as a point in B it rotates about ab, its virtual radius being $(ab)(bd)$; as a point in D it ro-

tates about ad, with a virtual radius $(ad)(bd)$. The following construction may be used for finding $V°D$ relatively to A. Take $(ad)F = V°B$, draw FH parallel to $(ad)(cd)$, and HL parallel to $(bd)(ad)$ intersecting $(ab)(cd)$ at N and $(ad)(cd)$ at L; then, by similar triangles,

$$\frac{HL}{HN} = \frac{(ad)(bd)}{(ab)(bd)} = \frac{V°B}{V°D}.$$

HN is therefore equal to $V°D$ on the same scale as that used for HL.

It should be noted that the constructions given above do not require that the centro be accessible. While there are many other constructions that are somewhat simpler for special cases, the ones given are applicable to all cases, and, for this reason, are the only ones herein considered.

Velocity Diagrams.

22. Reciprocating parts of engine.—When the link D of Fig. 11 is fixed, the mechanism is that of an ordinary steam or gas engine. The link D, enlarged sufficiently, becomes the frame or bed of the engine; then A is the crank, B the connecting-rod, and C the cross-head to which the piston and its rod are rigidly attached. The centre line of the main shaft is at ad, that of the crank-pin at ab, and that of the cross-head at bc.

Assuming that the angular velocity of the crank is constant throughout the entire revolution, which is approximately true, the linear velocity of the piston and the cross-head, it being the same for both, can be found for any position of the crank if the length of the crank and connecting-rod are known.

In Fig. 20, A is the main shaft, B_1 the crank-pin, D_1 the cross-head pin, and C the centro of the connecting-rod when in the position B_1D_1.

Using the foot and minute as units, and calling the number of revolutions per minute N, gives for the linear velocity of B,

$$V\text{-}.B = 2\pi(AB_1)N.$$

Take B_1F_1 on any scale, equal to $V\text{-}.B$, and draw F_1E_1 parallel to B_1D_1; then, by similar triangles,

$$\frac{D_1E_1}{B_1F_1}=\frac{D_1C}{B_1C}=\frac{\text{Virtual radius of } D_1}{\text{Virtual radius of } B_1}.$$

And since $B_1F_1 = V\text{-}.B_1$, therefore $D_1E_1 = V\text{-}.D_1$.

(Linear velocities are proportional to virtual radii.)

By taking a second position of the crank, as AB_2, and the corresponding positions of the other moving parts, the same con-

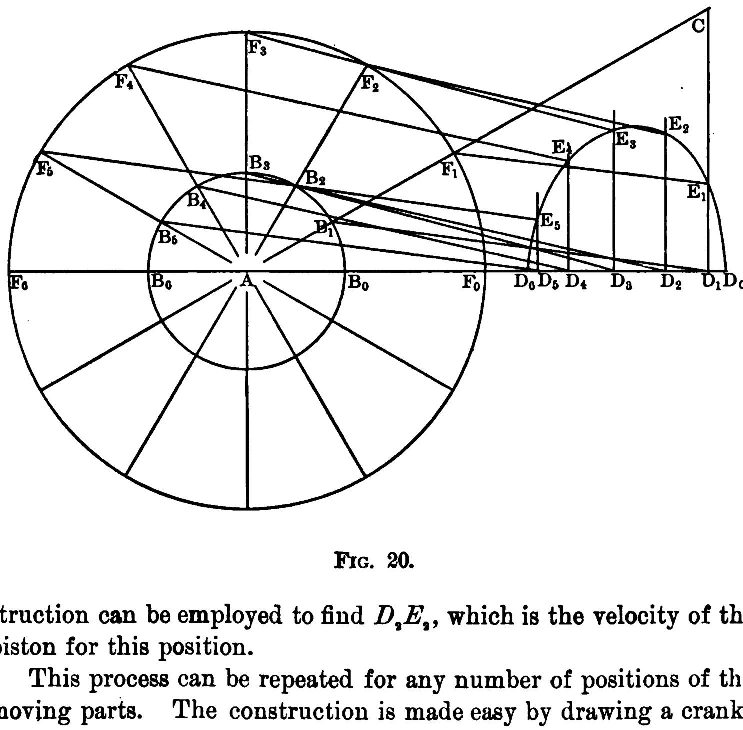

Fig. 20.

struction can be employed to find D_2E_2, which is the velocity of the piston for this position.

This process can be repeated for any number of positions of the moving parts. The construction is made easy by drawing a crank-pin circle of radius AB_1 about A as a centre, and, concentric with it, a circle of radius $AB_1 + B_1F_1$; then, taking any number of positions B_1, B_2, B_3, etc., on the crank circle, the radial lines drawn through these points give F_1, F_2, F_3, etc. Now take B_1D_1, B_2D_2, B_3D_3, etc., as the connecting-rod lengths, erect perpendiculars to AD_1 at D_1, D_2, D_3, etc., and draw F_1E_1, F_2E_2, F_3E_3, etc., parallel

to B_1D_1, B_2D_2, B_3D_3, etc., thus obtaining the points E_1, E_2, E_3, etc. A smooth curve drawn through these points gives the velocity diagram for the single stroke of the piston from one end to the other of the cylinder. If the complete diagram for the double stroke is drawn, it will be symmetrical about AD_0 when the velocities for the forward motion and those for the return are taken on opposite sides of AD_0. This being the case, it is necessary to plot only the diagram for the motion of the piston in one direction.

By making use of the piston velocities found above, a polar velocity diagram, showing the relative velocities of the piston and crank-pin, can be laid out as follows: Draw a circle, Fig. 21, of a

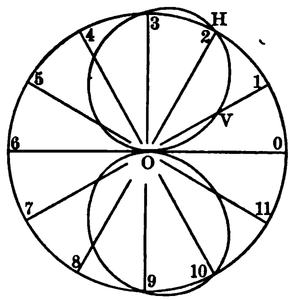

FIG. 21.

radius equal to the crank-pin velocity B_1F_1, Fig. 20, and divide it into parts corresponding to the positions of the crank in Fig. 20. Upon each position of the crank, measure from the centre of the circle the corresponding velocity of the piston as found in Fig. 20, and draw a curve through the points thus located. Thus, $O1$ and OV, Fig. 21, represent respectively the position of the crank AB_1, and the corresponding piston velocity D_1E_1, of Fig. 20.

In Fig. 21 it may be seen that the piston velocity exceeds that of the crank for a portion of the stroke, while at H and 3 they are the same. This occurs at H when the triangle $(ab)(bd)(bc)$, Fig. 11, is isosceles, the sides meeting at bd being equal. Between H and 3, Fig. 21, the angle of the triangle at ab, Fig. 11, is greater than that at bc; consequently the virtual radius $(bd)(bc)$ is greater than $(bd)(ab)$; therefore the velocity of the piston is greater than that of the crank-pin. At 3 the crank is at right angles to the line of motion of the piston, which places the virtual centre of the con-

necting-rod at an infinite distance; hence the crank-pin and piston have the same velocity.

The polar diagrams for the forward and backward motions are symmetrical about the axis of the piston.

When the axes of the piston-rod and main shaft do not intersect, but lie as in Fig. 22, where A is the main shaft and DD' the line

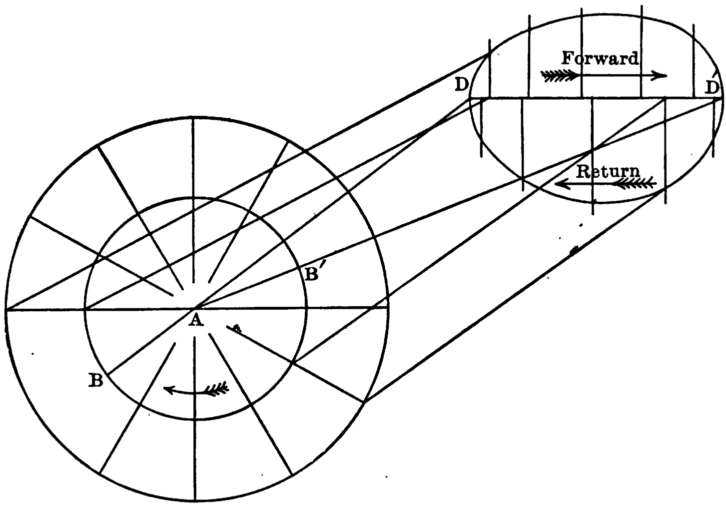

FIG. 22.

of piston travel, the velocity diagram is not symmetrical about DD', the forward motion being slower than the return.

This arrangement can be used for the "quick-return" motion of shaping-machines, where the cutting-tool is clamped to the ram, which corresponds to the piston of an engine.

The position BD of the connecting-rod at the completion of the return stroke is found by taking a radius $AD =$ length of the connecting-rod less that of the crank, and with A as a centre describing an arc cutting DD' at D; then, by drawing a line through D and A, the point B is obtained, which is the position of the crank-pin at the end of the return stroke; for a movement of the crank in either direction would bring B nearer to D, and, as the connecting-rod has a constant length, the end at D would move toward D'. Hence D must be at the end of the stroke.

The position B', at the end of the forward stroke, is found by taking a radius AD' = length of the connecting-rod plus that of the crank, and with A as a centre describing an arc cutting DD' at D'; then, by joining A and D', the point B is found.

23. Variable-motion mechanism.—Fig. 23. This mechanism is the slider-crank chain of Fig. 11 with the link A fixed. By rotating B at a uniform rate, D is driven at a variable velocity.

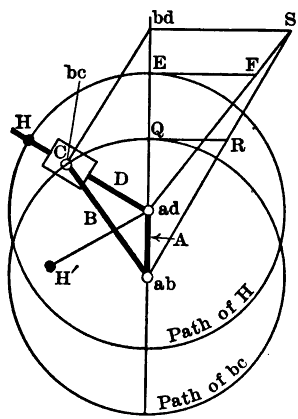

Fig. 23.

The velocity of bc may be either assumed, or readily calculated when the length of D and its angular velocity are known. The linear velocity of any point on D, as H, can be found by various methods, two of which will be given, the first by the use of centros, and the second without. Probably the second is somewhat more readily used practically.

1st method.—Taking the positions of the parts as in Fig. 23, draw a circle of radius $(ab)(bc)$ about ab as a centre. This gives the path of bc. Another circle of radius $(ad)H$, drawn about ad as a centre, is the path of H. Draw $(bc)(bd)$ perpendicular to D at bc, cutting A extended at bd. Then bd, being a point common to both B and D, must have the same linear velocity in both. To find the velocity of bd, draw QR perpendicular to A and of such a length as will represent the given velocity of bc. Then draw $(ab)R$, extending it to intersect a line drawn normal to A at bd.

This gives $(bd)S = V\text{-}.bd$ in accordance with the fact that the linear velocities of any two points in a rigid body (in this case B) are proportional to their radial distances. The triangles $(ab)QR$ and $(ab)(bd)S$ are similar.

Having thus obtained the linear velocity of a point, bd, in D, that of H is found by joining S and ad, and then drawing EF perpendicular to A and intersecting $(ad)S$ at F. This gives $EF = V\text{-}.H$.

2d method.—Fig. 24. The positions of the parts are taken the same as before, H being the same point on D. Draw $(bc)R$ normal to B, to represent the direction of motion and the linear velocity of bc. Draw $(bc)S$ perpendicular, and RS parallel to D. Then $(bc)S$

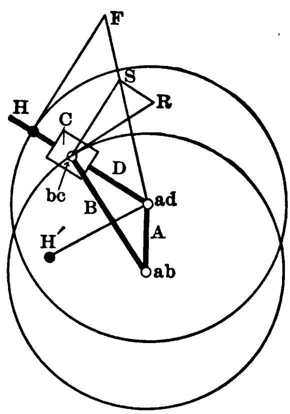

FIG. 24.

is the linear velocity of a point in D which is coincident with bc. Draw $(ad)S$ and extend it to intersect HF drawn perpendicular to D. Then $HF = V\text{-}.H$.

If, in either case, H is taken at the same radial distance as bc [*i.e.*, $(ad)H = (ab)(bc)$], the linear velocities of H and bc will be proportional to the angular velocities of their respective links.

The linear velocity of any other point in the link D, as H', bears the ratio $\frac{(ad)H'}{(ad)H}$ to that of H.

By taking several positions of the mechanism, and determining $V\text{-}.H$ for each, a polar velocity diagram can be plotted as shown

in the right-hand portion of Fig. 25, in which $(ad)F'$ corresponds to the position of D in Figs. 23 and 24, and $E'F'' = EF$ in Fig. 23 and $= HF$ in Fig. 24. The other radial lines represent successive positions of D, the corresponding velocities of H being measured on them beyond the circle of H. A curve drawn through the points thus determined is the velocity curve of H.

The maximum velocity of H occurs when it is at E, Fig. 23, the links A, B, and D coinciding for this position. Its lowest velocity is reached when it has moved through a half-revolution from this position of maximum velocity.

The maximum and minimum volocities of H' lag behind those of H by an angle $H'(ad)H$.

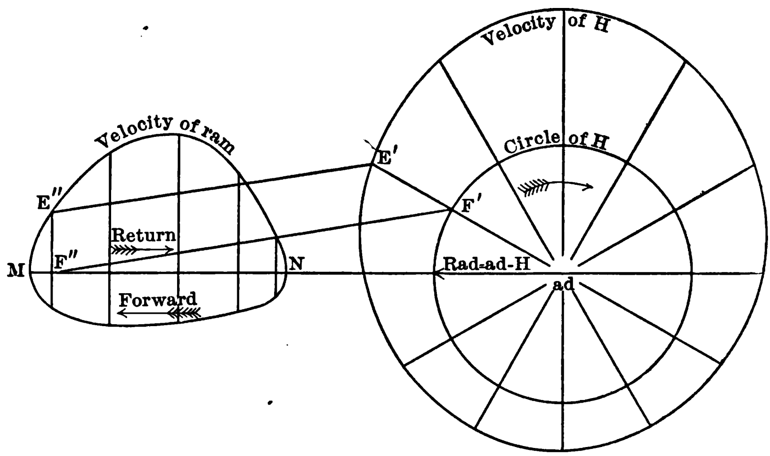

Fig. 25.

24. The Whitworth quick-return motion, Fig. 26, is obtained by adding to the variable-motion mechanism just described a connecting-rod E and sliding-piece F, as shown. The motion of F is horizontal and reciprocating. A is stationary.

The velocity diagram of F is found in the same general way as that for the mechanism of Fig. 20. Fig. 25 illustrates the method to be used. The velocity curve of H has already been found. $F'F''$ corresponds to the connecting-rod HF of Fig. 26. The line of travel of F'' is along MN. By drawing a perpendicular to MN at F'', and $E'E''$ parallel to $F'F''$, intersecting the perpendicular

at E'', the distance $F'E''$ is obtained as the velocity of F'', which is the same as that of all parts of the slide F.

Continuing the same operations for different positions of the

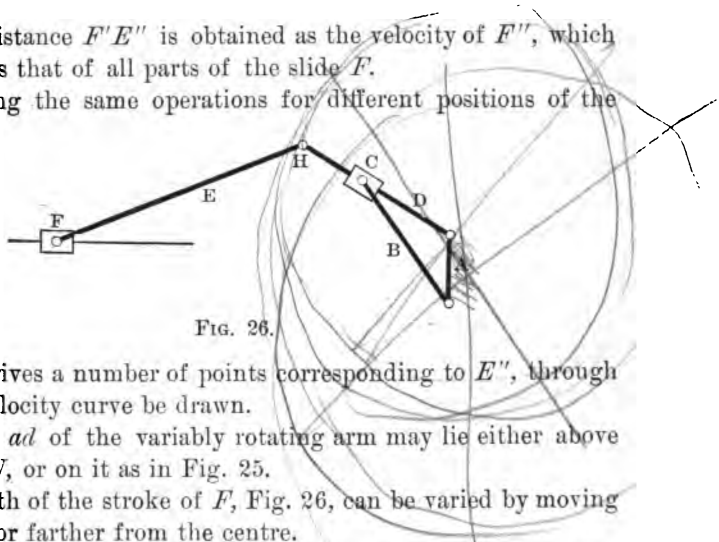

FIG. 26.

mechanism gives a number of points corresponding to E'', through which the velocity curve be drawn.

The axis ad of the variably rotating arm may lie either above or below MN, or on it as in Fig. 25.

The length of the stroke of F, Fig. 26, can be varied by moving H nearer to or farther from the centre.

25. Problem.—In the design of a shaper driven by the mechanism of Fig. 26, it is customary to assume a maximum stroke for the tool or ram, and a time ratio for the forward and return strokes. The driving parts are then designed to fulfill the assumptions.

In accordance with this, let MN, Fig. 27, be the length of stroke, the line MN being the path travelled by the axis of the pin joining

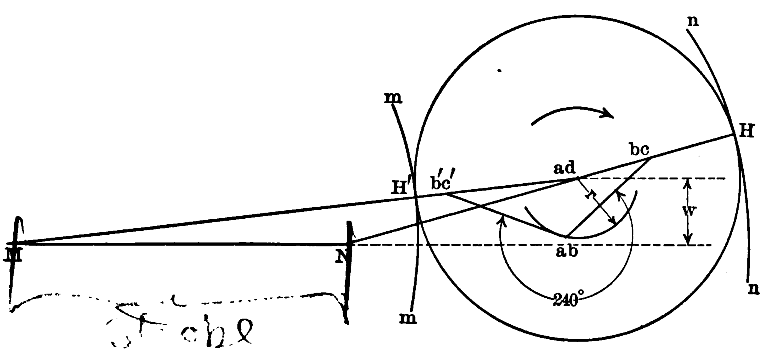

FIG. 27.

the connecting-rod to the ram. Also let the time of the forward stroke be double that of the return, which gives a time ratio of 2:1.

The length of the connecting-rod MH' can now be taken with

due regard to the length of stroke. With the length of the connecting-rod as a radius, and M and N as centres, strike the arcs mm and nn. The circle in which H, which is the axis of the pin joining the connecting-rod to the variably rotating arm, travels for the maximum stroke, must be tangent to both these arcs whatever the position of its centre ad. Assuming that ad is to be at a distance w above the line MN, it can be located by drawing a line parallel to and at a distance w above MN, and then, by trial, finding the point ad on this line, which is the centre of a circle tangent to mm and nn.

The positions, MH' and NH, of the connecting-rod at the ends of the stroke are found by drawing $(ad)M$ and $(ad)N$, the latter being extended to cut the circle at H.

During the forward stroke from N to M, the variably rotating arm travels, as indicated by the arrow, from the position $(ad)H$ to $(ad)H'$, swinging through something more than a half-revolution. The time for passing through this angle is to be, in accordance with the conditions of the problem, twice as long as that for the remainder of the complete revolution. This means that the uniformly rotating arm, corresponding to B in Fig. 23, must swing through two-thirds of a revolution, or 240°, while the variably rotating arm moves from $(ad)H$ to $(ad)H'$. The angle through which B swings for the return stroke is, of course, 120°.

There are now two methods for completing the solution. One is to assume a distance between the axes of the rotating arms, and the other to assume a length of the uniformly rotating arm. In either case the remaining proportions are determined to comply with the assumptions made.

First, suppose that r is taken as the distance between the axes of the two arms. Then ab, the axis of the driving-arm, must lie on the arc of a circle of radius r, Fig. 27, whose centre is at ad.

For convenience in solving the remainder of the problem, two lines may now be drawn at an angle of 120° on a piece of thin sheet celluloid.

The intersection of the lines on the celluloid can now be placed over any point on the arc of radius r, within the limits of practical working, as ab, and the celluloid so adjusted that equal lengths $(ab)(bc)$ and $(ab)(b'c')$ are cut on both lines as shown. These equal

distances represent the length of the driving-arm to be used to fulfill the conditions of the problem.

While any practicable position of *ab* on the arc of radius *r* will give the assumed time ratio, each will give a different velocity diagram for the ram.

For the second case, assume, in Fig. 28, the length of the driving-arm as $(ab)(bc) = (ab)(b'c')$, and mark this distance from the intersection on both lines on the celluloid.

Place the points just marked, over the lines *M*(*ad*) and *N*(*ad*) extended, as shown in the figure. By shifting the celluloid so as

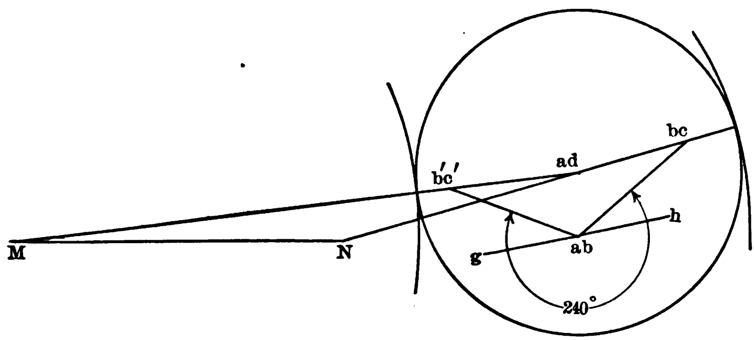

FIG. 28.

to move the points along the lines, *ab* will trace a curve *gh*. By fixing *ab* at any point on this curve, the required velocity ratio will be given to the forward and return strokes. The velocity diagrams for each position of *ab* will be different.

Naturally, in the design of such a machine as that just considered, the proportions will be selected by trial so as to give the most satisfactory results.

26. Oscillating-arm quick-return motion.—Fig. 29. The slider-crank mechanism of Fig. 11 is used for this motion, the link *A* being fixed and *B* the uniformly rotating driver. By taking *B* shorter than *A*, *D* is made to oscillate between the positions indicated by the two lines drawn from *ad* tangent to the circle of *bc*. A point *H* in *D* oscillates in an arc of a circle about *ad* as a centre.

The velocity of *H* can be found for any position of *D*, as fol-

lows: The linear velocity of bc is known or can be found if the angular velocity and length of B are known. To find the linear

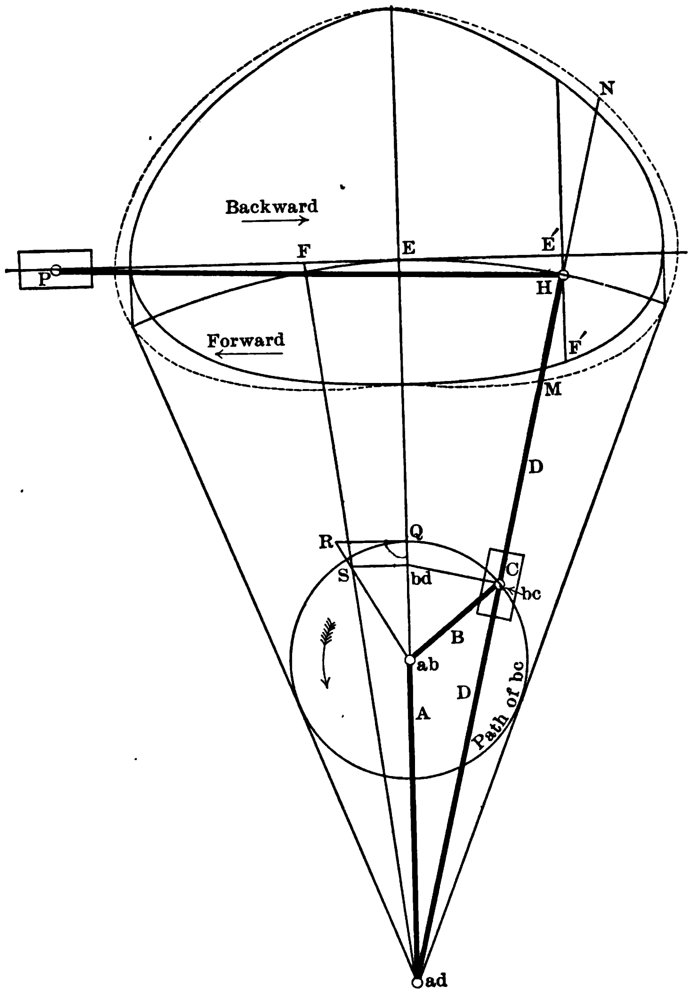

Fig. 29.

velocity of bd, take $QR = V\text{–}.bc$; join R and ab, and draw $(bd)S$ parallel to QR; then $(bd)S = V\text{–}.bd$. Since bd is a point in D, the linear velocity of H, which is also a point in D, is found by

drawing a line through E parallel to QR, and another through ad and S; the intersection of these lines at F gives $EF = V\text{-}.H$.

Two velocity diagram are shown in the figure. The one drawn with a full line has the straight line, EF, perpendicular to A, as a datum line. Points upon this curve are determined as follows: For the given position of the mechanism, the velocity of H has been found to be EF; through H draw a line perpendicular to EF and intersecting it at E'; then from E' take the distance $E'F' = EF$, thus obtaining F' as a point through which the velocity curve of the forward stroke must pass. The velocity of H when in the same position on the return stroke is taken upward from E' on the line HE', thus obtaining a point on the velocity curve of the return stroke. Other points are located in a similar manner.

The curve just determined probably appeals to the eye more readily than one laid off from the arc of a circle as a datum line; but if the velocity diagram of a sliding-ram connected to H by the link HP is to be found, a velocity curve having for a datum line the arc of the circle in which H oscillates is more easily used.

Points on this curve having the path of H as a datum line are determined as follows: From H take the distance $HM = EF$, on D, which gives M as one point on the curve of the forward stroke. By measuring upward on the line of D a distance $HN = V\text{-}.H$ when in this position on the return stroke, a point N on the curve for the return stroke is determined. All points necessary for drawing the velocity curve are found by similar operations.

The method of obtaining the velocity diagram of P, which is a point in the ram and has the same velocity as all other points in the ram, is almost the same as that for Figs. 20, 22, and 25, the only difference being that for each position of H two points on the diagram for P can be found, one on the forward and the other on the return stroke. Thus, for the given position of H, the corresponding point on the diagram for the forward stroke of P is located by drawing (construction not shown in the figure) through P a perpendicular to the line of travel of P, which in this case is PE, and then drawing through M a line parallel to HP; the intersection of the last line drawn and the perpendicular to PE is the point required. The point on the return stroke is found by draw-

ing a line through N parallel to HP, whose intersection with the perpendicular to PE through P is the one required.

27. Problem.—In the mechanism of Fig. 29, the length of the path through which P travels, and the time ratio of the forward and return strokes, are often given in practice, and the proportions of the various members made such as to fulfil these requirements.

In accordance with this, let MN, Fig. 30, be the given length of stroke, and the time ratio equal 5 : 3. The length of the connecting-rod may be taken as MH'. The right-hand end must, therefore, lie on the arc mm, described with M as a centre and radius

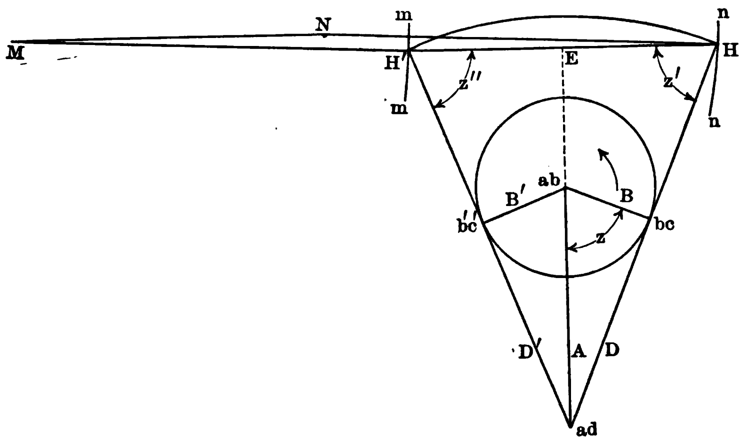

Fig. 30.

MH', when the ram is at the forward end of the stroke. At the end of the return stroke the right-hand end of the connecting-rod must lie on the arc nn, having the same radius and N as a centre. The points H' and H on these arcs are taken as the two extreme positions of the connecting-rod. The same points must, of course, be the extreme positions of the upper end of the oscillating arm.

By supposing, for a moment, that the problem has been completely solved, it can be seen that during the forward stroke B must swing in the direction of the arrow from the position B, normal to D, around to B', which is normal to D'; and for the return stroke, from B' to B, moving in the same direction of rotation.

Since B moves at a uniform rate, the time ratio of the two

strokes is proportional to the respective angles swung through for them.

By the aid of the similar triangles $(ad)(ab)(bc)$ and $(ad)HE$, it can be seen that the angles z and z' are equal. But z is half the angle passed through by B on the return stroke; therefore z' must be equal to half the angle passed through by B on the return stroke.

Returning now to the solution in accordance with the data given, it can be seen that for a time ratio of $5:3$, B must rotate through 225° on the forward stroke, and 135° while returning. The angle z' must be made, therefore, $135° \div 2 = 67.5°$, and z'' has the same value.

Drawing $H(ad)$ and $H'(ad)$ both at an angle of 67.5° with HH', locates ad at their intersection and determines the length of D. The axis ab, about which B rotates, may now be taken at any point on a line bisecting the angle $H(ad)H'$. By locating ab, the length of B is determined as the radius of a circle whose centre is at ab, and which is tangent to the extreme positions of the oscillating arm D

When used on a shaping-machine, the length of the stroke of the above mechanism is changed by varying the length of B, all other parts having constant dimensions. When B is made shorter, the time ratio approaches more nearly to unity, becoming almost equal to this value when B is very short.

CHAPTER II.

TOOTHED GEARS.

SPUR GEARS.

28. Pitch circles.—In the operations of machinery it is frequently necessary that two parts shall rotate about parallel axes with a constant relative angular velocity ratio, one driving the other.

In Fig. 31, A and B are two bodies that are to rotate about axes perpendicular to the paper, piercing it at ac and bc; C is the

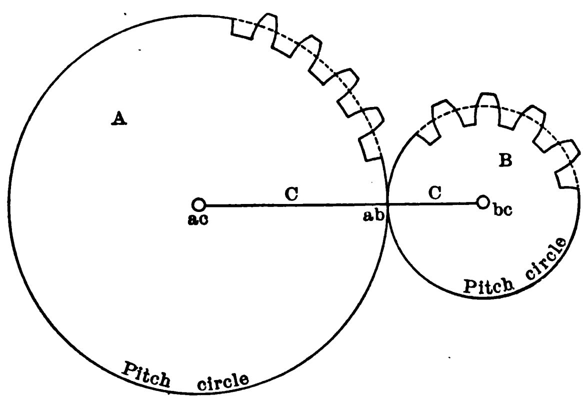

Fig. 31.

link joining A and B. Applying the principles that the three points ab, ac, and bc must lie in the same straight line (§ 8), and that the angular velocities of points having the same linear velocity are inversely as their radii (§ 19), the location of ab, which is a point common to both A and B, can be found by dividing $(ac)(bc)$ into two parts proportional to the required angular velocities of the bodies, so that

$$\frac{(ab)(bc)}{(ab)(ac)} = \frac{V^\circ A}{V^\circ B}.$$

Since the velocity ratio is to be constant, the centrodes of A and B are circles with centres at ac and bc. Therefore, by attaching two circular cylinders to A and B, of such diameters that they are in contact on a line projected at ab, the required velocity ratio could be obtained by rotating one as a driver, provided no slipping were to occur between the cylinders. In practice some slipping does take place between such surfaces, especially when power is transmitted, so it is found necessary to make intermeshing projections and recesses on the cylinders which engage with each other in such a manner as to prevent slipping. When this is done, the cylinders do not, of course, exist longer, but they are still considered as imaginary, and are called the "**pitch cylinders.**" The projections or sections of these imaginary cylinders are called "**pitch circles.**" The point of tangency of the pitch circles, ab, is called the "**pitch point.**"

29. Tooth curves for a constant velocity ratio.—When teeth are formed on the cylinders as described in the preceding paragraph, each becomes a "*spur gear.*" In order that the velocity ratio shall be absolutely constant, not only for complete revolutions, but at any instant, which means that a point on the pitch circle of one shall have exactly the same linear velocity as a point on the other pitch circle at any instant, it is necessary for the teeth to be formed with special regard to a constant velocity ratio. Gears that do not have the teeth so formed are not only noisy when running rapidly, but are unfit for use in some cases on account of producing vibration in the machinery or not giving the required uniform velocity ratio.

It can be seen from the above that, at whatever points on their curves a pair of teeth are in contact, the velocity ratio must be the same as when other points are in contact. That is, the velocity ratio must be the same as if the pitch cylinders were rolling together without slipping.

In Fig. 32, A and B are a pair of teeth of the gears whose pitch circles are in contact at ab. Since every point in B moves relatively to A about ab as a centro, the point d, where the teeth are in contact, considered as a point in B, must move at right angles to $d(ab)$. In order to permit this motion, the curve of A at this point must be perpendicular to $d(ab)$, that is, tangent to TT', which is at right angles to $d(ab)$. The motion of A relatively to B is also about ab

as a centro, and therefore d, as a point in A, moves along TT', which requires the curve of B to be perpendicular to $d(ab)$. The only motion that can take place between the teeth is, therefore,

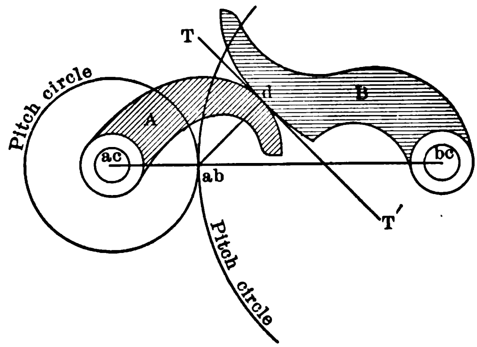

FIG. 32.

slipping at right angles to the line joining the point of contact, d, to the virtual centre ab.

The above conditions may be stated as follows: **In order for a pair of gear teeth to transmit a constant velocity ratio, the curves must be such that a perpendicular to them at their point of contact at any instant will pass through the pitch point.**

When a suitable curve has been selected for one tooth, that of the one to engage with it can be found by the method shown in Fig. 33, where A and B are the centres of two gears whose pitch point is P, and abe is a given tooth curve on A.

Cut two disks from thin, transparent sheet celluloid matted on one side, or tracing-cloth, to a radius somewhat less than AB, draw the pitch circle of A on one piece and pin it to the draughting-board at A, through the centre of the circle. Trace the given curve abe upon the disk, and draw perpendiculars to the curve at a and e, intersecting the pitch circle at g and h. Now draw the pitch circle of B on the remaining disk, and pin it to the board at B, so that the pitch circles are tangent at P. Graduate the pitch circles by lines uniformly spaced at equal distances apart on both. These graduations, when near together, can be used for turning the disks as if they were rolling together without slipping on each other.

Rotate the disks as if they were rolling together on their pitch circles, until g and g' coincide with P, and a lies at A'; then, since g has become the pitch point, and ga is perpendicular to the tooth curve, A' must be the point of contact between the given tooth and the one to be found. A point marked on B to coincide with A' when g and g' are both at P is, therefore, a point on the required curve. Rotate the disks again until b coincides with P; then b is the point of tooth contact, for it lies on the pitch circle. Mark b' directly over

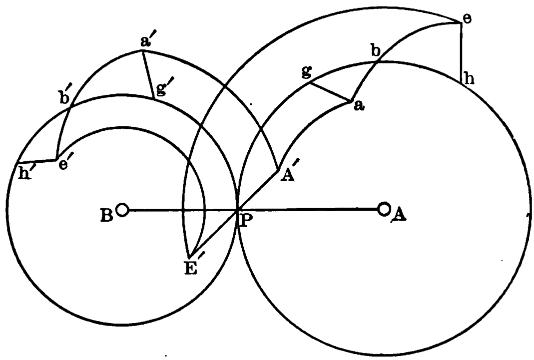

Fig. 33.

b as another point in the required curve. Rotate the disk a third time, so that h and h' lie at P, and e at E'; then e', coincident with e and E' for this position of the disks, is a third point on the curve.

A curve drawn through $a'b'e'$, perpendicular to $a'g'$ and $e'h'$, gives the approximate tooth outline. For greater accuracy, more points must be taken on the given curve and the corresponding points determined on the required curve.

However numerous the lines drawn normal to the tooth curve, none of them can intersect another between the curve and the pitch circle. Such an intersection would indicate a reverse bend in the tooth curve, and that the point of contact of the tooth with its mate moves back and forth along the curve, which is impossible. The point of contact must move through consecutive positions on the tooth curve, which must correspond to consecutive positions of the pitch point on the pitch circles.

While any of an exceedingly great number of tooth curves will

answer to transmit a constant velocity ratio, there are but few in practical use to any great extent. These are generally chosen with regard to the comparative ease with which they can be developed and constructed. A few of the more common ones will be described.

30. Cycloidal tooth curves.—In Fig. 34, let A, B, and D be three disks whose centres are in a straight line, P being their common point of tangency. Let their initial positions be such that a, b, and d coincide with P. By pressing the disks together so that

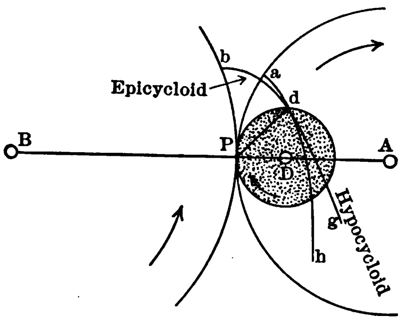

FIG. 34.

there can be no slipping between their curved surfaces, and rotating them about their centres in the directions indicated by the arrows, d will simultaneously trace the hypocycloid adg upon A, and the epicycloid bdh upon B. (There must necessarily be a piece of cardboard or similar material projecting beyond the disk B, in order to furnish a surface for tracing bdh.)

Since d traces the curves on A and B simultaneously, they must touch each other at a point coincident with d at any instant during their contact while in motion. Also, P is the centro of the motion of the disks relatively to each other at every instant. Therefore d moves, relatively to A and B, at right angles to Pd.

This is the condition required, according to § 29, for the transmission of a constant velocity ratio, namely : The normal to the tooth curves at their point of contact must pass through the pitch point.

NOTE.—It should be observed that if the rotation is the reverse of that indicated in Fig. 34, so that the point of contact approaches

the pitch point instead of receding from it, the conditions of a constant velocity ratio still remain true.

The describing circle D can have any diameter and the described curves will still be such that they will work together correctly. When its diameter is taken equal to the radius of A, d traces a straight line, which is a diameter of A. As the diameter of D increases further, the curve adg on A becomes convex on the side next to the centre of D and remains so until the describing circle is of the same size as A, when the curve is reduced to a point because there can be no rolling motion of D upon A. When the diameter of D is the same as that of A, the point d still traces an epicycloid on B. Therefore a pin without sensible diameter, attached to A at d, will engage with the epicycloid traced by itself upon B, and will transmit a constant velocity ratio. Practically, a pin of sensible diameter must be used. The tooth curve to engage with it for a constant velocity ratio can be found, as shown in Fig. 35, by

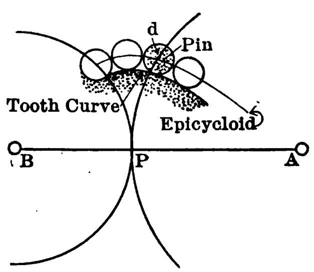

FIG. 35.

drawing a number of circles with centres on the epicycloid to represent several positions of the pin, and then drawing a tangent to them.

31. Double-curve teeth.—In Figs. 34 and 35 it can be seen that, as the point of contact recedes from the pitch point, the virtual radius Pd increases; therefore the slipping between the teeth increases. Also, if power is transmitted by one gear driving the other, the pressure between the teeth increases as the point of contact recedes from the pitch circles; for the pressure between the teeth is normal to their working surfaces at the point of contact, friction neglected, and its line of action passes through the pitch

point; therefore the lever-arms about the axes of the gears are shortened as *Pd* increases in length, and consequently the pressure between the teeth must increase in order to keep the torsional moment constant.

Owing to the above facts, only a small portion of the curves are used in practice in order to secure economy of power and durability of gearing by having a minimum amount of pressure and slipping between the teeth.

Let *bd*, Fig. 36, be the portion of the curve on *B* chosen to be

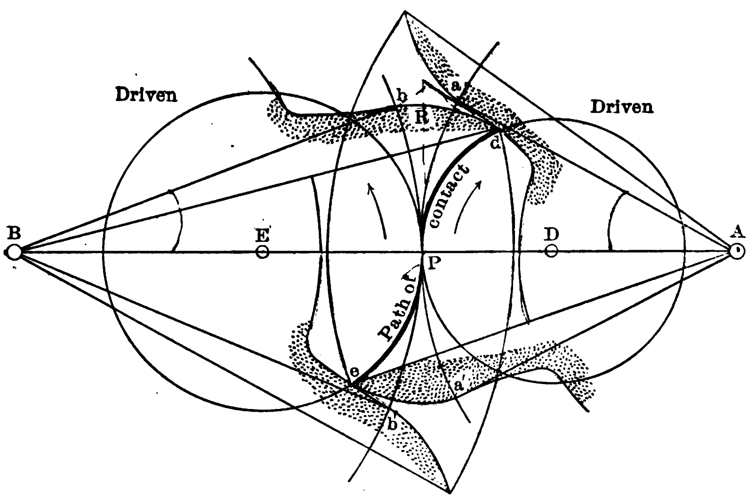

FIG. 36.

used; then *ad* is the part of the curve on *A* that will engage with it. Since *d* is the most distant working point on *bd*, measured from *B*, there is no necessity for the tooth on *B* to extend beyond a circle of radius *Bd*. The material of *A* must be cut away sufficiently to clear the point of the tooth *R*.

The curves *ad* and *bd* remain in contact while *A* rotates through the angle *PAa*, and *B* through the angle *PBb*. The arcs *Pa* and *Pb* are, of course, of equal lengths.

In order for the gears to continue their relative motion after passing through these angles, there could be another pair of curves placed as shown in the figure, so as to come in contact at *e* at the

instant of the separation of the first pair. This second pair of teeth would remain in contact while the gears rotate through the angle $a'Aa$ and $b'Bb$ respectively, and transmit a constant velocity ratio. (See note to § 30.) When a' and b' coincide with P, the conditions are the same as when a and b were at P; hence the second pair of teeth can be similar to those first considered in action, and so on for other pairs.

Teeth which work together as their point of contact recedes from the pitch point run more easily and quietly than when the point of contact approaches the pitch point. In order to have contact on only one side of the pitch point, the teeth would have to be made of single curves, those of one gear extending above the pitch circle, and those of the other lying below it.

In general practice, however, it is usually found advisable to use two curves meeting at the pitch circle to form each side of every cycloidal tooth. This is done, as shown in Fig. 36, by using two describing circles, D and E, the curves generated by D being ad and bd, and those by E, $a'e$ and $b'e$. By this means the distance between the working sides of the consecutive teeth on a gear can be made greater, and the portion of the curves used shorter, than when only the curves drawn by the describing point on one circle are utilized. Both sides of a tooth are usually made of the same outline so that the gears will run equally well in either direction and with either as driver.

Double-curve teeth are those having the working part of each side made up of two curves as described above. In such teeth the two generating circles can have either the same or different diameters, as best suits the case. Standard gears generally have both these rolling circles of the same diameter.

32. Definitions.—The angle through which a gear rotates while one of its teeth remains in contact with its mate on the gear that meshes with it, is the **angle of action.** For the gear A, in Fig. 36, the angle of action is $a'Aa$, and the corresponding arc, $a'Pa$, is the **arc of action**. The angle $a'AP$, passed through while the contact point is approaching the pitch point, is the **angle of approach,** and PAa, passed through while it is receding from the pitch point, is the **angle of recess.** The corresponding arcs are the **arc of approach** and the **arc of recess.** The distance, measured along the

pitch circle, between similar points of adjacent teeth, is the **circular pitch.** In practice, the circular pitch is about two-thirds of the arc of contact. **Diametral pitch** is the number of teeth per inch of diameter of the pitch circle of a gear. Thus, if a gear has 36 teeth and is 3 inches in diameter, its diametral pitch is $36 \div 3 = 12$. It can be seen that

$$(\text{Diametral pitch}) \,.\, (\text{Circular pitch}) = \pi.$$

The **addendum circle** bounds the ends of the teeth. The **working-depth circle** lies below the pitch circle a distance equal to that of the addendum circle above it; it does not indicate, however, that the sides of the teeth come in contact with their mates to this depth. The whole depth or **root circle** lies inside of the working-depth circle a distance equal to the clearance allowed for the points of the intermeshing teeth. The working surface of the tooth above the pitch line is the **face**; that below, the **flank. Backlash** is the difference between the thickness of a tooth and the space into which it meshes, measured on the pitch circle. On accurately cut gears the thickness of the teeth is almost exactly equal to the width of the space. In rough cast gears backlash must be allowed for irregularities.

Fig. 57, page 65, shows the ordinary form of teeth with names of parts.

In recent years there is a strong tendency toward the use of shorter teeth, especially when they are to perform heavy service. Several examples of large gears with short teeth are now giving excellent service in places where the more common proportions, as referred to above, have been unsuccessful.

33. Path of contact.—When the three disks in Fig. 34 rotate about their centres, the point *d* traces an arc of a circle on the draughting-board, which is the locus of the points of contact between the teeth; therefore the path of contact between cycloidal gear teeth is an arc of a circle.

When contact is on both sides of the pitch point, as in Fig. 36, the path of contact is composed of two arcs of circles meeting at the pitch point. In Fig. 36 the path of contact is along the arcs *eP* and *Pd*.

Fig. 37 shows double-curve gears in mesh. When *A* is the driver,

rotation being as indicated, contact is along $d'Pe'$; should B drive, contact would be along ePd. The length of the path of contact is

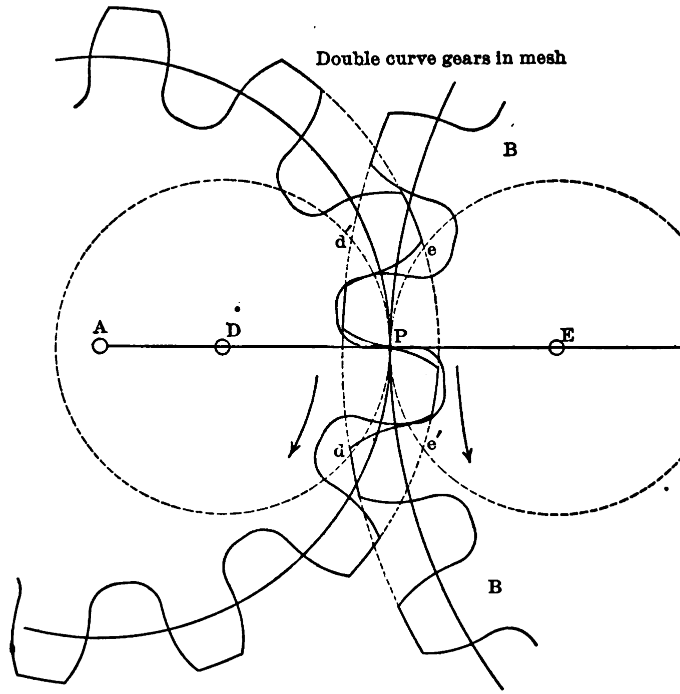

FIG. 37.

the same as that of the arc of contact, measured on the pitch circle of either gear.

34. Involute tooth curves. — In Fig. 38, a is a short cylindrical drum with a cardboard disk A attached to it; D is a straightedge with a tracing-point on its edge at d.

Suppose that D is placed tangent to a at T', as shown in the figure, and rolled about a without slipping, until d and the point of tangency coincide at T. During this motion d will trace an involute curve on A. At any instant during the motion, d moves about the tangent point as a centro. Therefore a line perpendicular to the curve at any point is a tangent to the drum; and, con-

versely, any tangent to the drum on the concave side of the involute intersects the curve at right angles.

The curve traced on A would have been exactly the same if the

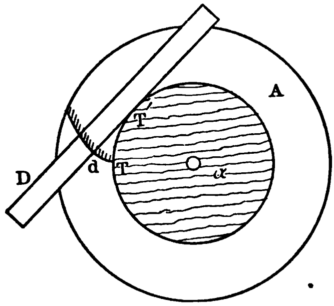

FIG. 38.

straight-edge had been moved in the direction of its length, causing the drum and disk to rotate about their common centre until d came into coincidence with T.

In Fig. 39, A and B are two disks whose pitch point is at P; gh is the edge of a strip of thin material corresponding to the tangent side of the straight-edge in Fig. 38, stretched across P and wound on the drums or base cylinders α and β, whose radii are pro-

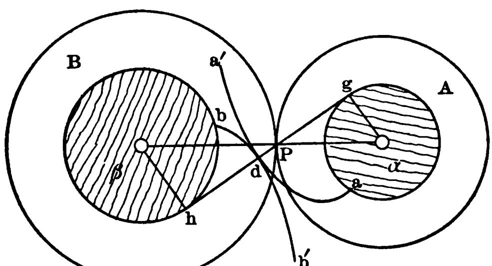

FIG. 39.

portional to A and B. If A and B are rotated without slipping on each other at P, the strip will unwind from one base cylinder and wind upon the other, always keeping taut, and a point d, on gh, will trace the involutes ada' and bdb' upon A and B respectively.

(The proof that gh will keep taut is as follows: P is a point

common to both A and B, and has the same linear velocity in each; a and b are points on the surfaces of the base cylinders. Since the linear velocities of points in the same body are proportional to their distances from the centre of rotation,

$$V\text{-}.g = \frac{g\alpha}{P\alpha} \,.\, V\text{-}.P$$

and
$$V\text{-}.h = \frac{h\beta}{P\beta} \,.\, V\text{-}.P.$$

Therefore
$$V\text{-}.g : V\text{-}.h = \frac{g\alpha}{P\alpha} : \frac{h\beta}{P\beta}.$$

But
$$\frac{g\alpha}{P\alpha} = \frac{h\beta}{P\beta}$$

Therefore
$$V\text{-}.g = V\text{-}.h,$$

which shows that the surface velocities of the two base circles are the same. This being true, the band will unwind as rapidly from one drum as it is taken up by the other.)

The point d traces the curves on A and B simultaneously; therefore they must be tangent at d. Also, gh is perpendicular to both curves at their point of contact and passes through the pitch point. These are the requirements of a tooth curve for a constant velocity ratio. (§ 29.)

The point of contact between the curves always lies on gh; therefore, for involute teeth, the path of contact is a straight line.

In practice the path of contact is generally taken at an angle of about 75° with the line of gear centres. This refers to the angle $gP\alpha$ in the figure.

It is a notable fact that, with involute gear teeth, the distance between the axes of a pair of gears can be changed, within the limits of practical working, without affecting the velocity ratio. This is due to the fact that changing the distance between the centres does not change the form of the involute curve so long as the base circles retain their original diameters. The only changes are in the obliquity of gh and the diameters of the pitch circles.

RACKS.

35. A rack is a straight bar, generally rectangular, with gear teeth cut on one or more sides of it. The pitch line is a straight line, and therefore its radius is infinite.

36. Cycloidal rack.—Fig. 40 shows the pitch circle *GH*, and addendum circle *IJ*, of a spur gear which is to mesh with a rack whose pitch line is *MN* and addendum line *KL*, the pitch point being at *P*.

D and *E* are the describing circles for the cycloidal teeth. When *A* is rotated and the rack moves toward the left without

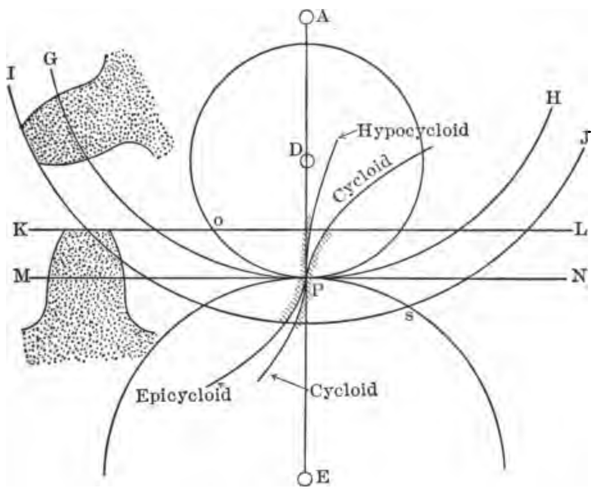

Fig. 40.

slipping between the pitch lines at *P*, a point on *D*, starting from *P*, will trace a hypocycloid on *A* and a cycloid on the rack at the same time. Also, a point on *E* will trace an epicycloid on *A* and a cycloid on the rack simultaneously. The curves traced at the same time will work together to transmit a constant velocity ratio, which, since the radius of the rack is infinite, must be the ratio of linear velocities. A point on the pitch circle of the gear moves with the same linear velocity as every point of the rack.

If the rack moves toward the left and drives the gear, contact

between a pair of teeth begins at *s*, follows the double-arc curve *sPo* and ends at *o*. Should *A* rotate counter-clockwise and drive the rack, contact would begin at *o* and end at *s*.

Since the rack is straight, it is impossible for motion to be continuous in one direction or "sense," and, as it must be reciprocating, it is necessary in many cases that the teeth be made so that driving can take place in both directions by either the gear or the rack. Teeth of the form shown in Fig. 57 are generally used.

37. Involute racks can also be made. In Fig. 41, *GH* and *MN* are the pitch circle and pitch line, and *IJ* and *KL* the addendum

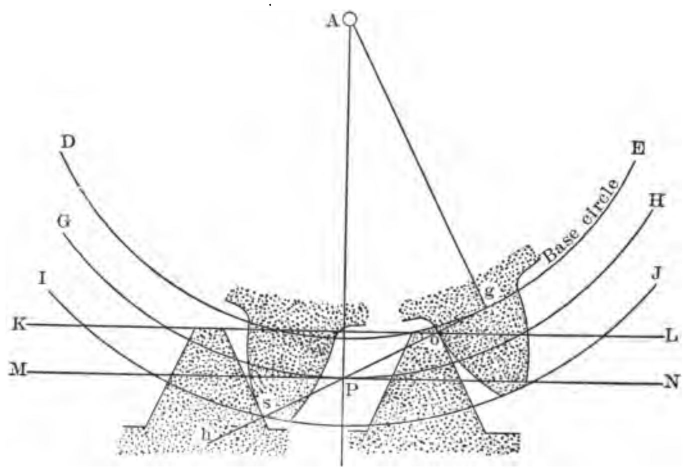

FIG. 41.

lines of the gear *A* and the rack. *DE* is the base cylinder of *A*. The radius of the base cylinder of the rack is infinite, therefore its involute is a straight line perpendicular to the line passing through *P* and tangent to the base cylinder of *A*. By rotating *A* and sliding the rack so that there will be no slipping at *P*, a point on the ribbon *gh*, will describe an involute on *A*, and a straight line on the rack, which will serve for tooth curves to transmit a constant velocity ratio.

With *A* driving clockwise, tooth contact will be along the line *oPs*, beginning at *o* and ending at *s*.

In practice the angle *gPA* is taken about 75° generally, for racks as well as spur gears.

ANNULAR GEARS.

38. When both centres of a pair of gears lie on the same side of the pitch point, the smaller is an ordinary spur gear, but the teeth of the larger must be cut on the inside of an annular ring of the material used. Such a gear is called an annular or internal gear.

39. Cycloidal annular gears.—The method of obtaining the curves of such a pair of cycloidal gears is shown in Fig. 42, where

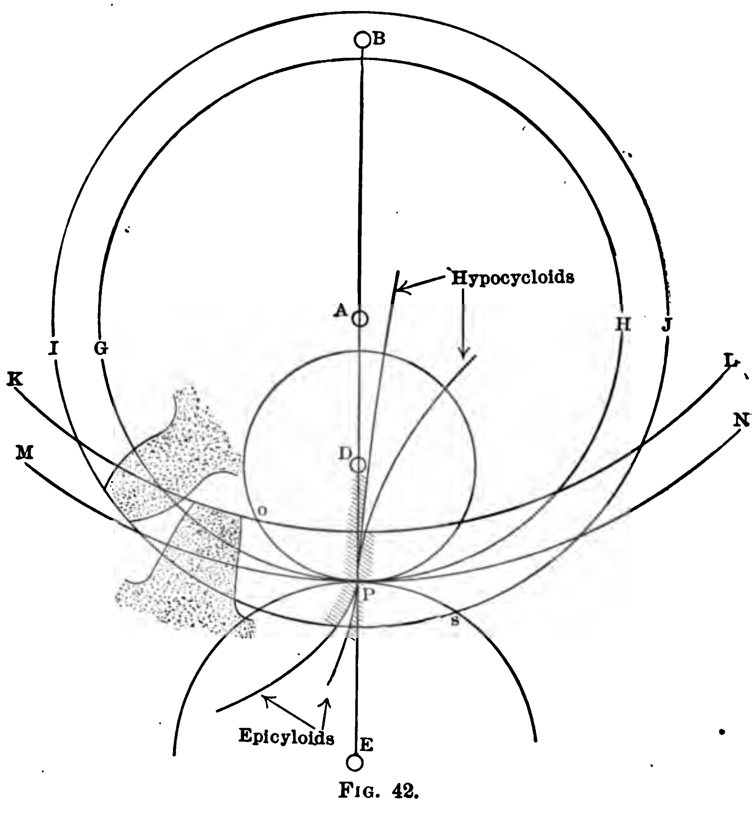

FIG. 42.

A and *B* are the centres of the spur and annular gears respectively; *D* and *E* are the centres of the describing circles; *GH* is the pitch circle and *IJ* the addendum circle of *A*; and *MN* is the pitch circle and *KL* the addendum circle of *B*.

When rotation takes place without slipping at the pitch point *P*, a point on *D* will simultaneously describe hypocycloids on both *A* and *B*, which will engage correctly for a constant velocity ratio. During the rotation a point on *E* will trace epicycloids on *A* and *B*, which will engage correctly for a constant velocity ratio. If *A*

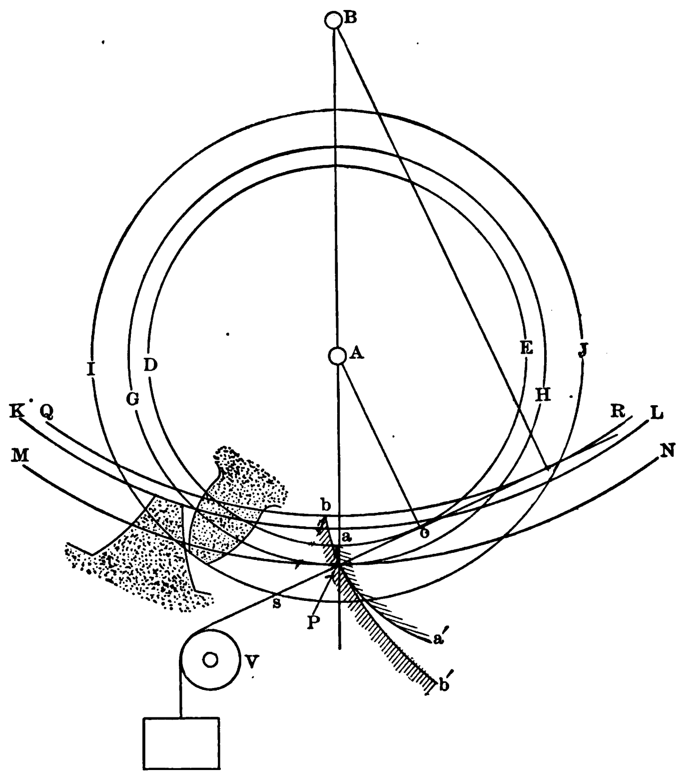

Fig. 43.

drives, rotating counter-clockwise, contact begins at *o* and ends at *s*.

The teeth of a cycloidal annular gear have exactly the same form as the spaces of a spur gear of the same pitch and pitch diameter, if no allowance is made for backlash.

40. Involute annular gears.—In Fig. 43, *A* and *B* are the centres of two gears whose pitch point is at *P*, that are to have in-

volute teeth. If the direction of the line of contact is given, it can be drawn through P, making the given angle BPo with the line of centres. The base circles (or cylinders) DE and QR are then drawn with centres at A and B, and tangent to the line of contact. Both points of tangency lie on the same side of P, so that a band or cord can not be used in the same way as in Fig. 39 to carry the tracing-point; but a straight-edge such as is shown in Fig. 38 can be placed tangent to both cylinders, and, when it is moved along without slipping on them, a point on the tangent edge will trace the proper involutes on the gear blanks.

Another method is to wrap a band or cord around one cylinder —either will answer—and carry it over a pulley V, Fig. 43, so that it passes through P, and attach a weight to the free end.

If the cord is wound on the large cylinder, or, if the straight-edge is used, the curves traced will be as shown in the figure, each starting from its base circle. But if the band is wound on the small cylinder, only that part of the involute of QR lying beyond a circle of radius Bo will be traced. Moreover, this is the only part of the involute bb' that is traced simultaneously with aa', whatever method is used; hence it is the only part of the curve that is useful. Therefore the teeth can be shortened and the arc of contact kept the same by making the addendum circle of B pass through o instead of occupying the position KL, as shown in the figure. Its correct position is through o. Contact begins at o and ends at s when A drives clockwise.

Two spur gears, meshing together, rotate in opposite directions. A spur and annular gear rotate in the same direction.

INTERCHANGEABLE SETS OF GEARS.

41. Cycloidal gears can be made so that any two or more of a set having the same pitch will mesh together properly. In order to do this they must be developed with the same diameter of describing circles. In Fig. 37, assume that the describing circles D and E have equal diameters. Now suppose A to be replaced by any other gear blank C, not shown. The tracing-points on the describing circles will trace suitable curves on C to mesh with B. Again, substitute C for B in the pair AB, and trace the curve on C that will engage

with the teeth of A properly. The curves traced on C when it replaces A are exactly the same as when it is substituted for B, because the same diameters of describing circles are used in both cases to describe both the hypocycloids and epicycloids; therefore C will mesh with either A or B. A fourth gear having teeth developed by the same describing circles will mesh with A, B, or C. And so on for any number of gears, including the rack and annular gears, the latter, however, having a limit as to smallness of diameter. Three or more such gears form an interchangeable set of gears.

42. Interchangeable involute gears must have the same pitch and a constant angle embraced between the line of centres and a common tangent to the base cylinders.

In Fig. 39, take gh as the given direction of the tangent to the base cylinders α and β of the gears A and B, whose pitch point is at P. A point d on gh traces involutes that work together properly for a constant velocity ratio. If A is replaced by any other gear C, tangent to B at P, and whose base circle is tangent to gh, the involute traced by d on C will engage properly with the involute of B. By putting C in place of B, its base circle will remain unchanged in size, hence its involute is unchanged, and is the correct one to engage with A for a constant velocity ratio. The same involute on C will, therefore, engage properly with both A and B. Since A, B, and C may have any number of teeth, any number of gears will mesh with each other when they have the same pitch and angle of obliquity.

The **angle of obliquity** is the angle between the common tangent to the base cylinders and a tangent to the pitch circles at P.

Laying out gear teeth.

43. Exact methods for laying out cycloidal teeth.—*1st method.*—Cut from a thin piece of wood two templates, as A and B, Fig. 44, one convex and the other concave on one side, both having curves of the same radius as the pitch circle of the gear upon which the teeth are to be developed. Also cut a disk of the same diameter as the describing circle selected, and attach a pencil to it so that the point is exactly on the circumference. The point should project slightly below the surface of one side of the disk, so that when the

latter is moved over the paper a line will be drawn by the pencil point.

Now draw the pitch circle, full size, on the paper, and divide it into as many equal parts as there are to be teeth in the gear. Take one of these points, as *P*, for the pitch point of one tooth curve. Place *A* so that one end of the concave side is at *P*, the curve of the template coinciding with the pitch circle above *P*. Lay the disk on the paper with the tracing-point at *P*, and then roll

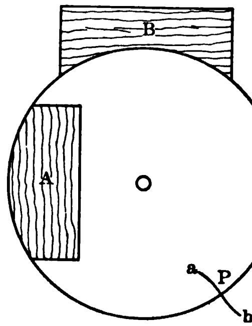

FIG. 44.

it along the convex side of *A*. By doing this the epicycloid *Pb* is drawn. Now remove *A* and place *B* so that one end lies at *P* and the concave edge lies on the pitch circle below *P*. Put the tracing-point of the disk at *P* and roll the disk along the concave side of *B*. This gives the hypocycloid *Pa*.

The two curves *Pa* and *Pb* form the outline of one side of a tooth.

By cutting a template to fit the portion of the tooth curve to be used, and pivoting it at the centre of the pitch circle, it can be swung about its pivot and brought successively into the positions for drawing the curves through the points previously marked on the pitch circle.

The thickness of the teeth can now be laid off from the curves just drawn, and if the teeth are to have the same form on both sides, as is ordinarily the case, the tooth-curve template can be turned over and pivoted to the centre again for drawing the remaining sides of the teeth.

If the teeth are to have different forms on their opposite sides,

the curves must be developed by the same method as the first ones, and another template brought into use.

2d method.—Another method which is more readily applicable in the draughting-room than the preceding one, and is probably as accurate when carefully executed, is as follows: Draw the pitch circle on the paper and the describing circle on the matted side of a thin piece of transparent celluloid. Place the circles tangent to each other, and pin them together with a fine needle stuck through the point of tangency and normal to the paper. Take a second needle and pierce the celluloid and paper with it where the separation of the two circles is just visible; it will be assumed that this is done on the right hand-side of the first needle. Remove the first needle and revolve the celluloid about the second until the circles are tangent on the right-hand side. Mark the paper by sticking the free needle through the first hole made in the celluloid. Then, with the same needle, pierce the celluloid and paper as before at the point of separation of the circles at the right of the needle sticking in the board. Remove the needle next to the starting-point, rotate the celluloid again, and puncture the paper by sticking the free needle through the first perforation in the celluloid. By continuing this operation, a number of points on the tooth curve are obtained, through which the epicycloidal portion of the tooth curve can be drawn. The hypocycloid is obtained in the same manner by putting the describing circle inside of the pitch circle.

By using very fine needle-points the motion of the two circles relatively to each other is only very slightly different from a true rolling motion, and with careful work the curves can be relied upon as being practically accurate.

44. Exact methods for laying out involute teeth.—*1st method.* First draw the pitch circle. This is represented in Fig. 45 by the circle of radius BP. Through any point P draw a radial line PB, and the line CD making a chosen angle β with it. With B as a centre draw a circle tangent to CD. This is the base circle for developing the involute.

Cut a template EF to fit the base circle on the concave side, and attach a fine wire to it near one end of the convex side. Attach a pencil point to the free end of the wire so that when the latter is wound on the curve of the template the pencil point will

lie against the curved side. Now place the template with the curved edge on the base circle and, keeping the wire taut, move the pencil point away from the template, thus tracing a curve on the paper which is an involute of the base circle.

If it is desired to have the involute pass through any particular point on the pitch circle, the pencil can be placed at this point, and the template adjusted accordingly.

The involute curve cannot extend farther below the pitch circle than the base circle. Hence, if the teeth are to be cut deeper

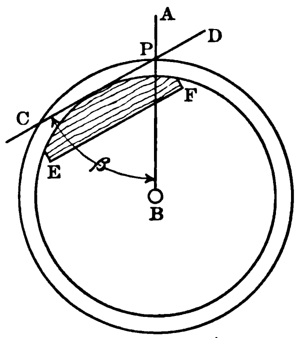

FIG. 45.

than this, the curves oelow the base circle need only be made of such a form as will clear the tops of the intermeshing teeth.

A straight-edge with a tracing-point on one edge can be used to roll on the template, instead of the wire.

2d method.—This is by the use of sheet celluloid, as in the second method of § 43. The base line is determined as in the preceding paragraph, and then a straight line is drawn on the celluloid and rolled along the pitch circle by the aid of needles, points on the tooth curve being determined by marking through a describing point on the straight line.

45. The Willis Odontograph is a convenient instrument for obtaining the approximate tooth outlines for drawings, and even for the actual teeth in some classes of rough work. Its adaptation depends on the fact that it is possible to find an arc of a circle that closely coincides with the true form of an involute tooth, and two arcs that give nearly the true form of the double-curve cycloidal tooth.

The form of the instrument adapted to involute teeth is shown in Fig. 46, this being the only form that is now manufactured. It consists of a piece of sheet metal cut to the form shown, the outer edges making an angle of 75° with each other, one of them being

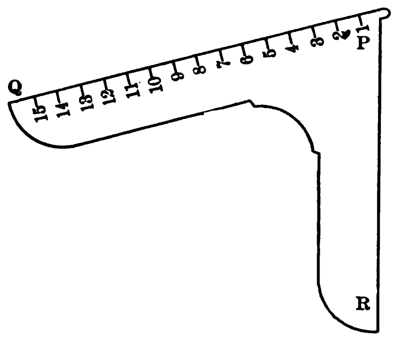

FIG. 46.

graduated in equal divisions, numbered according to a scale which is one-quarter size. Zero of the scale is at the angle, and the mark 1 is $\frac{1}{4}$ of an inch from zero, etc.

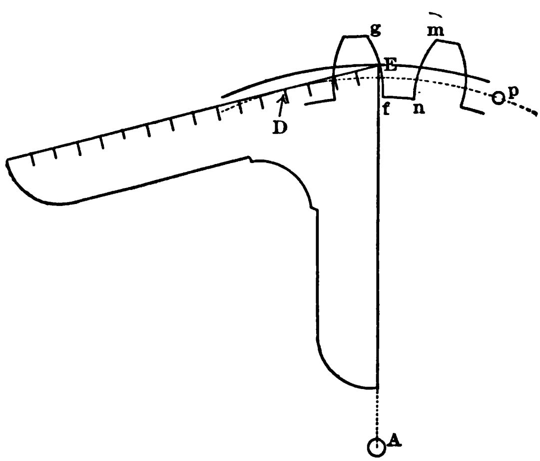

FIG. 47.

The directions for use accompanying the instrument are as follows (Fig. 47): "Let A be the centre and AE the radius of the

pitch circle of a proposed wheel. Lay the instrument upon the radius *AE* so that its point coincides with the pitch circle. The centre point will be found at once by reading off the radius of the wheel in inches upon the reduced scale. To describe the other teeth, draw with centre *A* and radius *AD*, a circle within the pitch circle, dotted in the figure. This will be the locus of the centres of the teeth; then, having previously divided the pitch circle, take the constant radius *DE* in the dividers, and keeping one point in the dotted circle, step from tooth to tooth and describe the arcs first to the right and then to the left; as for example, *nm* is described from *p*, and *fg* from *D*."

46. The Robinson Odontograph, Fig. 48, can be used to lay out tooth curves which almost exactly coincide with the true form.

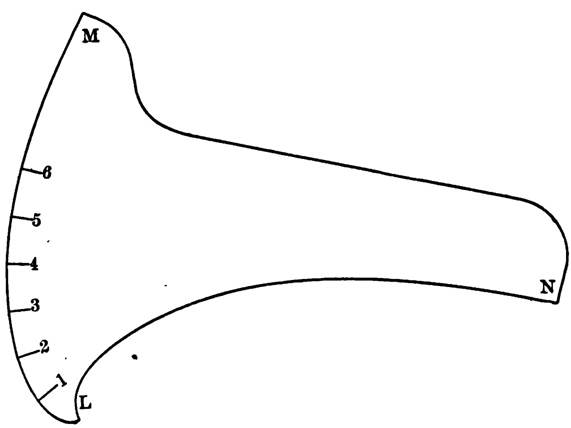

FIG. 48.

The two curves *LM* and *LN* are both of the same form; they are logarithmetic spirals, one being the evolute of the other.* The instrument is adapted to both cycloidal and involute gears, there being six tables of settings for as many "systems" of gears, including interchangeable and annular gears.

The following is taken from Professor Robinson's description of the instrument:

"In planning this odontograph, the leading endeavor was to

* Van Nostrand's Science Series, No. 24.

make its practical application the most convenient possible without sacrifice from accuracy of form of tooth obtained by it.

"It should be made of metal, because it is intended that the instrument, when desired, may be used directly as a template, in which use it will be subject to wear from passes of the scribe. It has several holes, so that it may be attached to any convenient wooden rod in such a manner that when the rod swings around a centre pin to the wheel all the faces of the teeth may be described directly from the instrument itself. The desired result is thus obtained directly, without intervening counterpoints and dividers.

"To place the instrument in position for drawing a tooth-face, a table is used which should accompany the instrument. From this

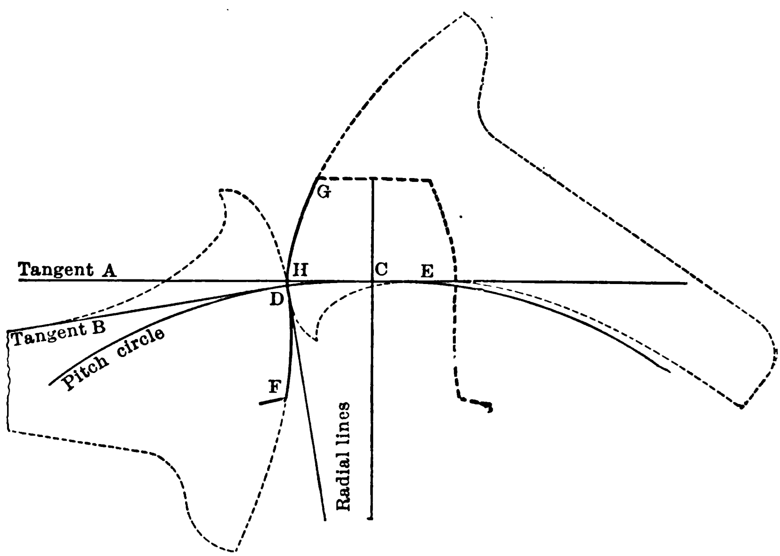

FIG. 49.

table a value is taken which depends upon the diameter of the pair of wheels and the number of teeth in the wheel for which the teeth are sought. This tabular value, when multiplied by the pitch, is to be found on the graduated edge *LM* of the odontograph. This done, draw the tangent *A*, Fig. 49, to the pitch line at the middle point, *C*, of the tooth, and lay off the half thickness, *CH* or *CD*, of the tooth on either the tangent or the pitch line. Then place

the graduated edge of the odontograph at *H*, and in such a position that the number and division of the scale found as above shall come precisely on the tangent line at *H*. Also get the curved edge *LN*, Fig. 48, so that the curve will just lie tangent to the tangent *A*, as at *E*. Then all is ready for tracing the curve for the tooth face from the pitch line through *D* toward *G* as far as needed. By turning the instrument, which is graduated on both sides, over, and doing likewise, we get the opposite face of the same tooth."

For the flanks of cycloidal gears the instrument is located in the same general way, it being swung around through something like 180° in its own plane and brought into position, with regard to a tangent *B* to the pitch circle at the pitch point *D* of the tooth-curve, as shown in Fig. 49. The flank *DF* is then drawn along the edge of the instrument. Different settings of the graduated edge are required for the face and flank.

47. Walker's method of laying out gear teeth by circular arcs. —In 1871 a "wheel-scale" for designing the teeth of wheels was patented by John Walker. It consists of a chart which gives to a full-size scale the necessary dimensions for proportioning teeth ranging in pitch from the smallest practicable size up to six inches. The face and flank of the tooth are separate arcs of different radii, the two arcs joining at the pitch point.†

48. Laying out gear teeth by rectangular coordinates.—Professor J. F. Klein of Lehigh University has computed and tabulated the values of the rectangular coordinates of tooth curves, the origin being taken at the pitch point. By this method "no describing pitch or any other circle need be drawn, thus doing away with all the preliminary draughting-room work and its possibilities of error."

The tabulated values are used to locate points on the tooth curve, through which the curve is afterward drawn.‡

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* See directions accompanying the instrument.

† These charts are supplied by John Walker, 576 West Adams St., Chicago, Ill.

‡ Klein's "Elements of Machine Design," second edition, 1892.

Cutting Spur and Annular Gears.

49. General methods.—By using, in connection with a proper spacing device, a tool which will cut a groove of the form of the space between two teeth, gear teeth can be formed.

The simplest method is to use a tool of the form shown in Fig. 50. This is such a tool as is ordinarily used in a metal-working planer or shaper, except that the outline *ABCD* is made the same as the spaces of the gear to be cut. The face *F* is normal to the elements of the teeth which the tool forms. While cutting the

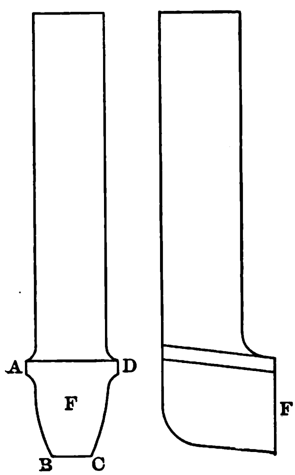

Fig. 50.

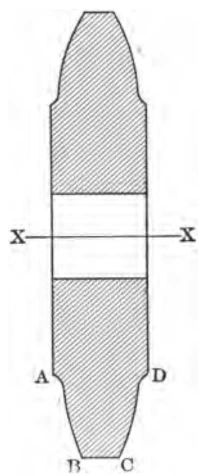

Fig. 51.

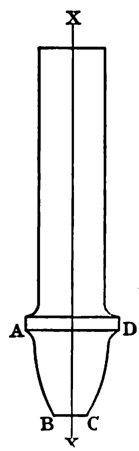

Fig. 52.

gear the tool is driven with a reciprocating motion across the face of the gear blank, the feeding of the tool into the work being downward as the material is removed, until the proper depth of space is finally reached. The blank can then be turned to the position for the next space and the operation repeated.

The rotary milling-cutter shown in axial section in Fig. 51, *XX* being the axis of rotation, is the most commonly used of all cutters for spur gears. It is made by turning the edge of a disk of steel to the form *ABCD*, corresponding to the tooth space of the gear to be cut. It is then fluted or grooved to form teeth with cutting edges upon it, such as are used on any milling-cutter. It is

used by rotating it about its axis, and at the same time feeding it across the face of the gear blank. A space can thus be formed by a single passage of the cutter across the blank. After this the gear is rotated to bring it into position for the next space, and so on.

The end milling-cutter, Fig. 52, is made by cutting the tooth-space curve *ABCD* while rotating the cutter-blank about the axis *XX*. The curved surface is then fluted to form the cutting teeth common to milling cutters. The flutes are not shown in the figure. This cutter is used by rotating it about its axis and feeding it across the face of the gear blank. The whole space can be cut out by a single passage of the cutter across the blank if desired.

The three forms of cutters, Figs. 50, 51, 52, have their special advantages for certain classes of work. Thus that of Fig. 50 is the only one that can be used on an annular gear having the sectional form of rim shown in Fig. 53, where *T* is the tooth and *H* is an

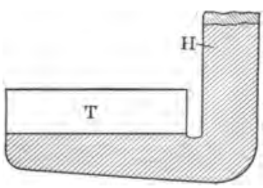

FIG. 53.

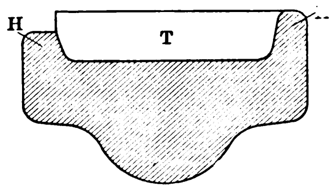

FIG. 54.

annular ring or an arm of the gear, there being only a narrow groove between this and the end of the tooth.

The end mill is the only one applicable to shrouded gears of the sectional form of Fig. 54, where *T* is the tooth and *H* is the shroud extending either partly or wholly to the top of the tooth, as the case may be. The ends of the spaces will have, of course, the form given them by the cutter.

50. Cutting interchangeable gears with sets of cutters.—For a given pitch the form of the space between the teeth changes with every change of the number of teeth in the gear. In a set of interchangeable gears, therefore, there are no two gears having the same form of space (unless there are duplicates).

In order to cut a set accurately by any of the methods of the preceding paragraph, it would be necessary to have as many cutters as there are diameters of gears. It is found in practice, however,

that sufficient accuracy can be attained by cutting several gears, ranging in diameter between certain limits, with a single cutter. These limits are generally given in terms of the number of teeth in the gears, for by this means they are applicable to all pitches, and the range for one cutter depends upon the degree of change of curvature as the number of teeth changes.

The following tables show the cutters adopted by the Brown & Sharpe Mfg. Co.

There must, of course, be a set of cutters for each pitch to be cut.

CYCLOIDAL CUTTERS. 24 Cutters in each set.				INVOLUTE CUTTERS. 8 Cutters in each set.	
Cutter.	Teeth.	Cutter.	Teeth.	Cutter.	Teeth.
A cuts	12	M cuts	27 to 29	1 cuts	135 to rack
B "	13	N "	30 " 33	2 "	55 " 134
C "	14	O "	34 " 37	3 "	35 " 54
D "	15	P "	38 " 42	4 "	26 " 34
E "	16	Q "	43 " 49	5 "	21 " 25
F "	17	R "	50 " 59	6 "	17 " 20
G "	18	S "	60 " 74	7 "	14 " 16
H "	19	T "	75 " 99	8 "	12 " 13
I "	20	U "	100 " 149		
J "	21 to 22	V "	150 " 249		
K "	23 " 24	W "	250 or more		
L "	25 " 26	X "	rack.		

An examination of the preceding tables shows that three times as many cutters are required for the cycloidal as for the involute gears. This is due to the fact that the forms of the teeth and spaces change more rapidly in the former than the latter. One of the causes for this is the fact that the describing circle is the same size for every gear of a cycloidal set, while the base circle of the involute is proportional to the number of teeth.

51. Cutting interchangeable sets of gears by conjugate methods.—It is possible to cut a complete set of interchangeable gears with a single cutter of either type shown in Figs. 50, 51, and 52. The cutter in this case must have the form of the tooth of some one of the set to be cut, instead of being like the space, as for the preceding methods of cutting.

Suppose that the type of cutter selected is that of Fig. 50, the outline of the face being that of a section of a tooth on any one of the set of gears. This cutter can be so mounted that, while it has a reciprocating motion across the face of the blank to be cut, it can be gradually moved relatively to the blank in the same way that a tooth of the gear to which it corresponds would move when meshing with the finished gear made from this blank. The reciprocating motion of the cutting tool will remove the material from the gear blank as it is brought before it by the gradual motion just mentioned. By removing the material in this manner the correct outline of the space is cut out.

Fig. 55 may aid in following the formation of the space by this operation. In this figure the cutting tool conforms in outline to the

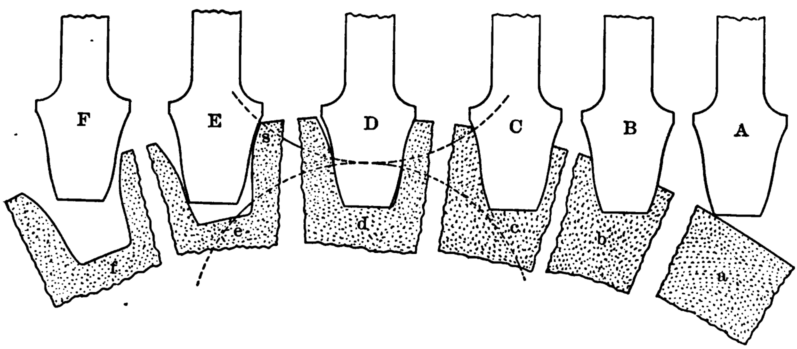

FIG. 55.

tooth of a cycloidal rack having a describing circle of a diameter equal to the radius of a fifteen-tooth gear of the same pitch as is to be cut. Such a tool can be fastened in the tool post of a shaper and gradually fed along at right angles to its cutting stroke. The blank to be cut can be mounted on a spindle attached to the frame or table of the machine. Some means of rotating the blank so that its pitch circle will move with exactly the same velocity as the feeding of the tool must be provided. This being done, the mechanism is complete, and one space can be cut by feeding the tool along so that if it starts from the position A it will successively pass through the positions B, C, D, E, and F. The corresponding positions of the part of the gear near the tool are indicated by the small letters.

Fig. 56 shows several stages of cutting an involute gear space by the same process on a shaper. The edges of the face of the cutter in this case are straight lines at an angle of 75° with the direction of feeding the tool.

It can be seen that in the latter case the corner of the tool cuts into the material of the gear beyond the smooth curve formed by the side, thus leaving the tooth narrow and weak at the bottom. This **interference** is sometimes so great as to cut away a considerable portion of the working part of the tooth curve. It can

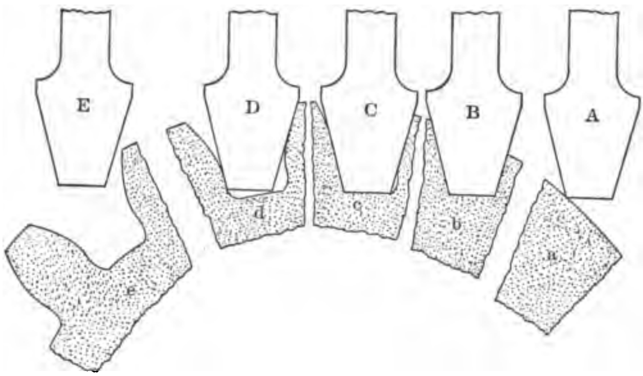

FIG. 56.

be overcome by making the addendum of the cutter or rack tooth shorter, or by rounding its corners.

In Fig. 55, where there is no interference, the tool has finished the space by the time it reaches the position *E*, the last material being removed at *s*, where the describing and addendum circles intersect, this being the end of the path of contact. The working surfaces of the teeth are always finished by the time the tool has just passed the end of the path of contact, but interference may continue past this position.

The above principle of conjugating the teeth has been practically applied in special machines where rotary instead of reciprocating cutters are used.*

* U. S. patent 327037 to Ambrose Swasey; U. S. patent 405030 to George B. Grant. Robinson's "Principles of Mechanisms," 1896.

Proportions of Gear Teeth.

52. The following tables (pp. 66–69) give the proportions of gear teeth as adopted by the Browne & Sharpe Mfg. Co., and almost universally used in this country for the smaller sizes of teeth, and to a considerable extent for the larger ones. Fig. 57 shows the parts named in the table.

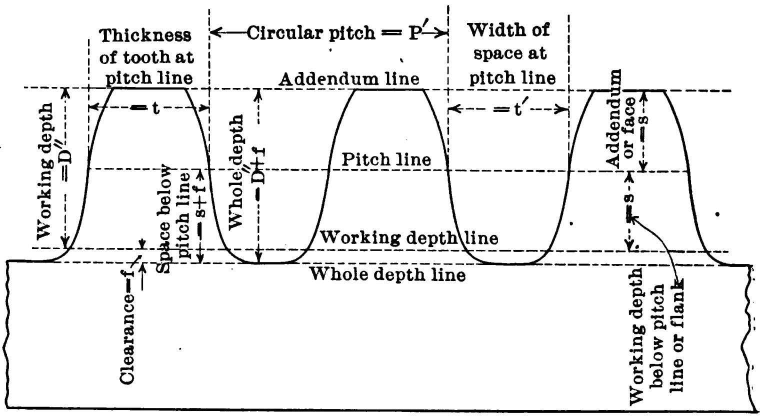

FIG. 57

In recent years there is a strong tendency toward the use of very much shorter teeth for the larger pitches especially. They are found to be stronger and to run more smoothly than the longer ones.

Mr. C. W. Hunt gives the following proportions of gear teeth as used by the Company with which he is associated. It refers to cast-iron gears moulded to form, and used without machining.*

The notation is:	The proportions are:
P = circular pitch in inches.	$A = .2P$.
A = addendum.	$D = .2P$.
D = dedendum.	$C = .05(P + 1)$.
C = clearance.	

A pair of large steel spur gears, 6-inch pitch, having about the same proportions as given above, are in service at the power-house

*Trans. Amer. Soc. Mech. Eng., vol. XVIII.

GEAR WHEELS: TABLE OF TOOTH PARTS.

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch. P	Circular Pitch. P'	Thickness of Tooth on Pitch Line. t	Addendum or $\frac{1''}{P}$. s	Working Depth of Tooth D''	Depth of Space below Pitch Line. $s+f$	Whole Depth of Tooth. $D''+f$
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
2¼	1.3963	.6981	.4444	.8888	.5143	.9587
2½	1.2566	.6283	.4000	.8000	.4628	.8628
2¾	1.1424	.5712	.3 36	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
3½	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	.0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541

GEAR WHEELS: TABLE OF TOOTH PARTS—*Continued.*

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch. P	Circular Pitch. P'	Thickness of Tooth on Pitch Line. t	Addendum or $\frac{1''}{P}$. s	Working Depth of Tooth. D''	Depth of Space below Pitch Line. $s+f$	Whole Depth of Tooth. $D''+f$
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360

GEAR WHEELS: TABLE OF TOOTH PARTS.

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch. P'	Threads or Teeth per inch Linear. $\frac{1''}{P'}$ C	Diametral Pitch. P	Thickness of Tooth on Pitch Line. t	Addendum or $\frac{1''}{P}$. s	Working depth of Tooth. D''	Depth of Space below Pitch Line. $s+f$	Whole Depth of Tooth. $D''+f$	Width of Thread-Tool at End. $P' \times .31$	Width of Thread at Top. $P' \times .335$
2	$\frac{1}{2}$	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6200	.6700
$1\frac{7}{8}$	$\frac{8}{15}$	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5813	.6281
$1\frac{3}{4}$	$\frac{4}{7}$	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5425	.5863
$1\frac{5}{8}$	$\frac{8}{13}$	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5038	.5444
$1\frac{1}{2}$	$\frac{2}{3}$	2.0944	.7500	.4775	.9549	.5525	1.0299	.4650	.5025
$1\frac{7}{16}$	$\frac{16}{23}$	2.1855	.7187	.4576	.9151	.5294	.9870	.4456	.4816
$1\frac{3}{8}$	$\frac{8}{11}$	2.2848	.6875	4377	.8754	.5064	.9441	.4262	.4606
$1\frac{5}{16}$	$\frac{16}{21}$	2.3936	.6562	.4178	.8356	.4834	.9012	.4069	.4397
$1\frac{1}{4}$	$\frac{4}{5}$	2.5133	.6250	.3979	.7958	.4604	.8583	.3875	.4188
$1\frac{3}{16}$	$\frac{16}{19}$	2.6456	.5937	.3780	.7560	.4374	.8156	.3681	.3978
$1\frac{1}{8}$	$\frac{8}{9}$	2.7925	.5625	.3581	.7162	.4143	.7724	.3488	.3769
$1\frac{1}{16}$	$\frac{16}{17}$	2.9568	.5312	.3382	.6764	.3913	.7295	.3294	.3559
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3100	.3350
$\frac{15}{16}$	$1\frac{1}{15}$	3.3510	.4687	.2984	.5968	.3453	.6437	.2906	.3141
$\frac{7}{8}$	$1\frac{1}{7}$	3.5904	.4375	.2785	.5570	.3223	.6007	.2713	.2931
$\frac{13}{16}$	$1\frac{3}{13}$	3.8666	.4062	.2586	.5173	.2993	.5579	.2519	.2722
$\frac{3}{4}$	$1\frac{1}{3}$	4.1888	.3750	.2387	.4775	.2762	.5150	.2325	.2513
$\frac{11}{16}$	$1\frac{5}{11}$	4.5696	.3437	.2189	.4377	.2532	.4720	.2131	.2303
$\frac{2}{3}$	$1\frac{1}{2}$	4.7124	.3333	.2122	.4244	.2455	.4577	.2066	.2233

GEAR WHEELS: TABLE OF TOOTH PARTS—*Continued.*

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch. P'	Threads or Teeth per inch Linear. $\frac{1''}{P'}$	Diametral Pitch. P	Thickness of Tooth on Pitch Line. t	Addendum or $\frac{1''}{P}$. s	Working Depth of Tooth. D''	Depth of Space below Pitch Line. $s+f$	Whole Depth of Tooth. $D''+f$	Width of Thread-Tool at End. $P' \times .31$	Width of Thread at Top. $P' \times .335$
$\frac{5}{8}$	$1\frac{3}{5}$	5 0265	.3125	.1989	.3979	.2301	.4291	.1938	.2094
$\frac{9}{16}$	$1\frac{7}{9}$	5.5851	.2812	.1790	.3581	.2071	.3862	.1744	.1884
$\frac{1}{2}$	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1550	.1675
$\frac{7}{16}$	$2\frac{2}{7}$	7.1808	.2187	.1393	.2785	.1611	.3003	.1356	.1466
$\frac{2}{5}$	$2\frac{1}{2}$	7.8540	.2000	.1273	.2546	.1473	.2746	.1240	.1340
$\frac{3}{8}$	$2\frac{2}{3}$	8.3776	.1875	.1194	.2387	.1381	.2575	.1163	.1256
$\frac{1}{3}$	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1033	.1117
$\frac{5}{16}$	$3\frac{1}{5}$	10.0531	.1562	.0995	.1989	.1151	.2146	.0969	.1047
$\frac{2}{7}$	$3\frac{1}{2}$	10.9956	.1429	.0909	.1819	.1052	.1962	.0886	.0957
$\frac{1}{4}$	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0775	.0838
$\frac{2}{9}$	$4\frac{1}{2}$	14.1372	.1111	.0707	.1415	.0818	.1526	.0689	.0744
$\frac{1}{5}$	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0620	.0670
$\frac{3}{16}$	$5\frac{1}{3}$	16.7552	.0937	.0597	.1194	.0690	.1287	.0581	.0628
$\frac{1}{6}$	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0517	.0558
$\frac{1}{7}$	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0443	.0479
$\frac{1}{8}$	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0388	.0419
$\frac{1}{9}$	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0372
$\frac{1}{10}$	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0310	.0335
$\frac{1}{16}$	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0194	.0209

of the Chicago City Railway Company at Twenty-first Street. They replaced cast-iron gears of much larger pitch, which failed under the same work.*

STEPPED, TWISTED, AND HELICAL GEARS.

53. When spur gears wear with use, the teeth lose their correct outline and are apt to become noisy. This can be obviated in a measure by replacing the single wide-faced gear with two or more thin ones side by side, so arranged that the teeth of each succeeding one shall be in advance of the preceding one by an amount equal to the circular pitch divided by the number of gears used to form the complete gear. This is the method of making a **stepped gear.**

By increasing these thin gears to an infinite number, the steps between the teeth are reduced to zero, and the teeth assume a helical form. The gear thus becomes a short section of a many-threaded screw. Such a gear is called a **helical** or **twisted gear,**† a pair of which are shown conventionally in Fig. 58. The latter name is applied because the gear may be assumed to be made by twisting one end of an ordinary spur gear about its axis while the other end is held stationary.‡ In a rack the tooth elements become straight lines running diagonally across the face. In helical gears the angularity of the helices forming the tooth elements is generally greater than would be formed by twisting an amount equal to the circular pitch.

If a line AB is drawn on the pitch cylinder normal to the tooth elements which lie on the cyclinder, it will be the **normal helix** to the teeth. The **normal pitch** is the distance between similar points of consecutive teeth, measured along the normal helix. It can be seen that the normal pitch is less than the circular pitch.

If energy is transmitted by the gears, there will be a tendency for the teeth to slide over each other axially on account of their angularity. This causes an end thrust on the shafts, which is

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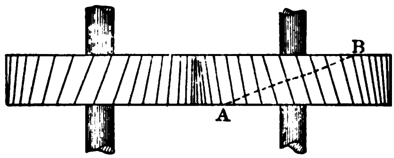
* *Street Railway Journal,* Feb. 1896, page 121.

† The name "spiral gear" is quite commonly applied.

‡ It can readily be seen that twisting may distort the teeth into any other form than helices, but on account of the great difficulties in making any but this form, the general case is neglected.

generally objectionable. It can be obviated by placing two gears of opposite angularity of teeth on each shaft, as shown in Fig. 59. Gears of this form run very quietly, and give excellent service under heavy as well as light loads.

FIG. 58.



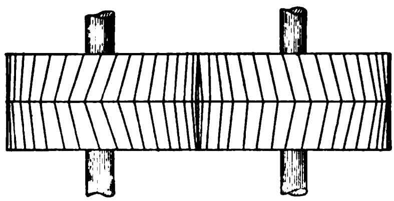


FIG. 59.

54. Cutting helical gears.—The teeth of helical gears can be cut with theoretical accuracy by a planing tool such as is used for ordinary spur gears. Such a tool is illustrated in Fig. 50 and described in § 49. In order to cut the teeth, the gear and tool are mounted as for cutting a spur gear. The blank, instead of being held stationary as the tool passes across it, however, must be rotated about its axis at a rate bearing a constant ratio to the velocity of the cutting tool. Each point in the cutting edge of the tool will thus be made to cut out a helical element of the tooth. The conjugate method described in § 51 can also be used by giving the gear an additional rotation during each stroke of the cutter. While these operations are theoretically simple, their practical application involves comparatively complicated and expensive machinery.

Approximate methods, giving teeth which closely approach the true from, are commonly used in practice. When the angularity of the tooth helix is not great, the rotary milling-cutter, Fig. 51, gives fairly accurate results. It is so set that its axis is parallel, or approximately so, with the direction of the normal helix where it in-

tersects the common perpendicular to the axes of the gear and cutter. The thickness of the cutter at the pitch surface must not be greater than the normal pitch.

The tooth space corresponding to the normal pitch will not be the same as that of the axial section of the rotary cutter which forms it. This is due to the fact that the finishing part of the cut is not taken in the axial plane of the cutter, but partly in advance of and partly behind this plane, a small portion being taken in the plane, of course. The result is a tooth space and tooth which do not conform accurately to the true outline. The deviation becomes greater as the angularity of the tooth helix increases.

The end milling-cutter of Fig. 52 gives very nearly the true outline when the angle of the tooth helix is small. The axial section of the cutter should be about the same as that of the tooth space corresponding to the normal pitch. This cutter also takes the finishing cut partly before and partly behind its axial plane tangent to the normal helix passing through the axis of the cutter.*

BEVEL GEARS.

55. Determination of pitch surfaces.—In Fig. 61, OA and OB are two shafts intersecting at O. They are to be connected by gearing so as to have an angular velocity ratio of $a:b$. The pitch surfaces must first be determined. To do this, draw a line parallel to OA and at a distance b from it, and another line parallel to OB and at a distance a from it Through the intersection of these lines at s, draw the line Os of indefinite length.

If two right cones with circular bases be placed with their apexes at O, and their axes coincident with those of the given shafts, the proportions being such that they will be in contact along os, they will roll together to transmit a constant velocity ratio. The motion will be a purely rolling one, for if any point P be selected on the common element of the cones, the distances of P from the two axes will have the ratio $a:b$, which shows that the ratio of the radial

* For further discussion of the methods of cutting helical gears, see "Practical Treatise on Gearing," by Brown & Sharpe Mfg. Co.; *American Machinist*, Nov. 21, 1885, and May 19, 1888; "Treatise on Gears," by George B. Grant, 1893.

distances of all points in Os is the same. If slipping occurs at any point, it will necessarily occur at all other points, and will be proportional to the distance from the apex. The entire cones or any pair of frustra of them in contact along any part of the tangency line can therefore be used.

In order to transmit energy without changing the velocity ratio, teeth must be provided on the gears as is done for spur gearing.

56. Cycloidal tooth curves for bevel gears.—In Fig. 60, ABC and ADE are two cones in contact along the element AP (not

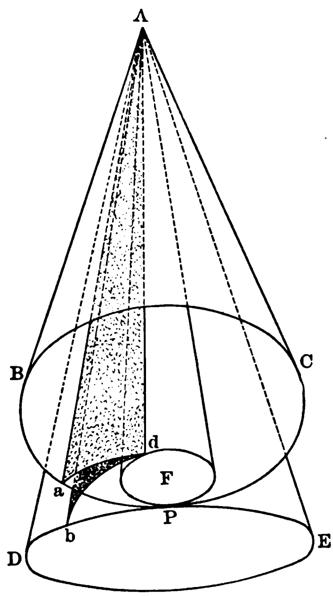


Fig. 60.

drawn), their common apex being at A. These may be taken as the pitch cones upon which cycloidal teeth are to be developed. ABC may be considered as made of an infinitesimally thin material, so that a third cone AF, with vertex at A, can lie against both the pitch cones along AP, being inside of ABC and outside of ADE.

If the axes of the three cones are kept in the same position, and they are rotated without slipping over each other, starting from an initial position such that a, b, and d coincide with P, d will

trace the spherical hypocycloid *ad* on the spherical base of *ABC*, and a spherical epicycloid *bd* on the extended or projecting spherical base of *ADE*.

The element *Ad* will at the same time sweep out the surfaces *Aad* in *ABC* and *Abd* on *ADE*. These two surfaces or any portions of them swept out simultaneously and lying at the same distance from *A*, may be taken to form the faces of teeth for transmitting a constant velocity ratio. The proof is similar to that for cycloidal spur gears. The two surfaces are formed simultaneously, and the instantaneous axis of the three cones is always at *AP*. The swept out surfaces must always be in contact along the element *Ad*, and a plane passed through *AP* and *Ad* will be normal to the surfaces.

For curves upon the opposite sides of the pitch cones, a describing cone must be rolled outside of *ABC* and inside of *ADE*, contact being along the element *AP*, as before.

57. Involute tooth curves for bevel gears.—In this case a disk having a radius equal to the height of the pitch cones can be placed with its centre at *A* and at any given or assumed angle with the plane of the axes of the cones. The angle corresponds to the 75° angle for spur gears. Two base cones coaxial with the pitch-cones and tangent to the disk are then used for the latter to rest upon.

By turning all the cones together and allowing the disk to be rolled along by the base cones, a point on the edge of the disk will trace spherical involutes on the spherical bases of the pitch cones, and a radial line of the disk will sweep out surfaces on the pitch cones, which will serve to transmit a constant velocity ratio if used for the tooth curves. Any portion of these surfaces intercepted by two spherical surfaces concentric with the apexes of the pitch cones, can be used in practice.

58. Laying out bevel gears.—Since the curve passed through by every point in the describing line for bevel gear teeth is a spherical one, it is impossible to develop it on the plane surface of a piece of drawing-paper. Some approximate method must therefore be used. Tredgald's method is sufficiently accurate for practice, and is the one commonly applied, possibly is the only one ever used. It is given in Fig. 61, where portions of the spherical bases

of the pitch cones are shown by the stipple shading. The true tooth-curves are traced upon these surfaces when they are generated by the method just described. If a conical surface BPF of some very thin material is placed tangent to one of these pitch-cones, as OPF, at its base circle PF, the curve cut in this surface by the generating line will differ but very slightly from that on the spherical surface for the portion of the curve that is used for teeth

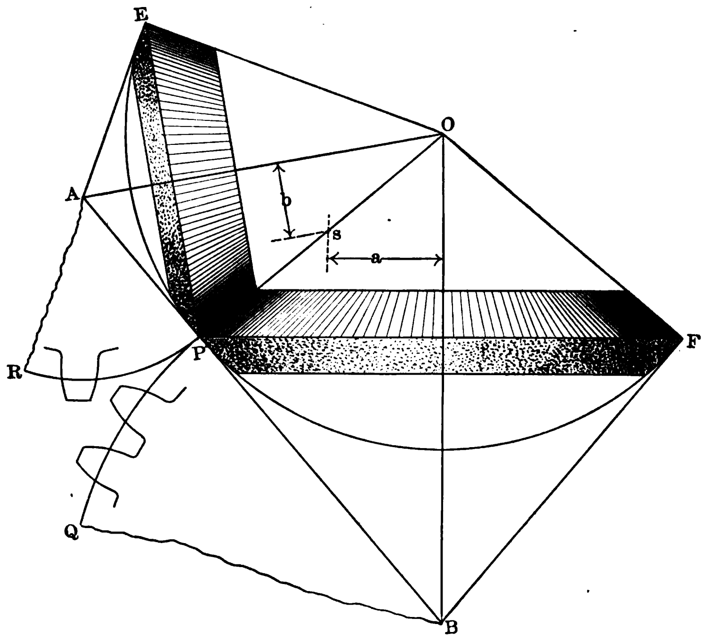


FIG. 61.

of the ordinary proportions. The conical surface BPF can be developed on the plane surface of the drawing-paper by cutting it down any element and allowing it to unroll. If it is cut down the element BP, the line which was tangent to the base of the pitch-cone will develop in a circle with B as a centre, a portion of which is shown as PQ, and the teeth will develop as shown. The same process can be applied to the pitch cone AEP.

In order to lay out the large end of the teeth by this method when the pitch cones are determined, AB is drawn perpendicular

to OP, intersecting the axes of the gears at A and B. The last two points are then used as centres for describing the arcs PR and PQ of indefinite length, only enough of them to allow the development of a few teeth being required. Then, using these arcs as portions of pitch circles, the teeth are developed upon them as is done for spur gears, according to any system that may be selected.

It must be kept in mind that the pitch and number of teeth must be such that the required number will go on a pitch circle of the diameter PF or PE.

Fig. 62 shows a partial section of the common form of a pair of

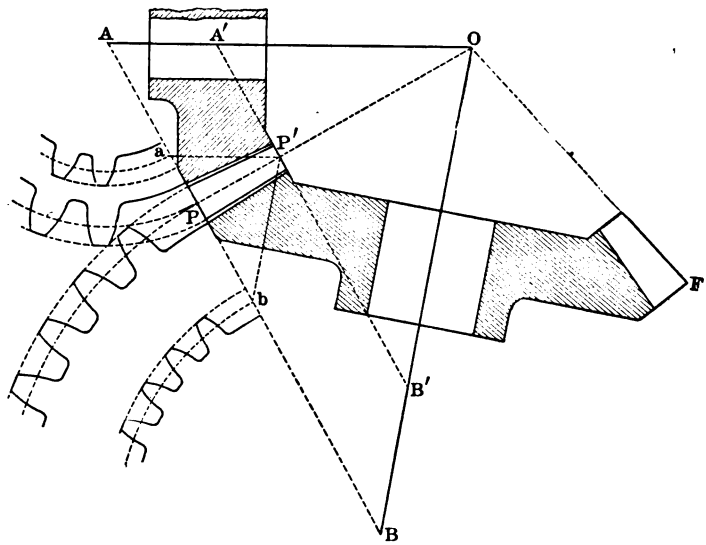


FIG. 62.

bevel-gear blanks and the method of showing the development of both ends of the teeth. If the method used for the larger end were followed exactly for the smaller one, A' and B' would be used as centres for the inner ends. For convenience and a clearer way of comparing the two ends, the distance Aa is taken equal to $A'P'$, and A is then used as a centre for the auxiliary pitch circle on which the teeth of the inner end of the gear arc developed. The same method is applied to the other gear of the pair.

In Fig. 62, POB is the pitch-surface angle, and FOB the face

angle of the larger gear. The same names are sometimes applied to the angles between the elements of these cones, which are double the angles named.

59. Cutting bevel gears.—Since the space between the teeth of a bevel-gear changes in dimensions constantly in going from one side of the gear face to the other, it is clearly impossible to cut the teeth by a single passage of a rotary or other cutter across the face, for the cutter can be made to conform to the space at only one place.

Approximately accurate cutting of small bevel-gears with rotary cutters is commonly done in practice by using special cutters. For any given gear the cutter is made to have its curves conform to those of the space at some selected part. Sometimes the space at the large end of the teeth is selected, and at other times the space profile one-third of the way across the face from the large end. The cutter must be made at least as thin as the width of the space at the small ends of the teeth, in order that the latter shall not be cut away when the cutter is fed across the face of the gear. Different methods of cutting are practised, the nature of the work being such that no one method can be said to have any great advantage over the others. The general method is as follows: The gear and cutter are mounted on the gear-cutting machine so that the cutter can have either the part of the cutting edge which forms the pitch point of the tooth or the part which reaches to the working depth of the tooth fed toward the apex of the pitch cone. A roughing cut is then taken across the face to remove the greater portion of the material to be cut away. The gear is then rotated about its axis and moved sidewise so as to bring it into such a position that the cutter will give as nearly the correct form to one side of the tooth from end to end when fed across the face as can be obtained by trial. The gear is then rotated in the opposite direction and shifted so that the other side of the cutter will give the same form to the opposite side of the space. This operation is performed for all the spaces, the result being a gear that will run fairly well with its mate, but which is only approximately correct in its tooth forms.*

*" Experimental Investigation of the Cutting of Bevel Gears with Rotary Cutters," by Forrest R. Jones and Arthur L. Goddard in Trans. Amer. Soc. Mech. Eng., vol. XVIII.

Accurately cut bevel-gear teeth of any system which has a straight line passing through the apex of the pitch cone for a generatrix can be planed to form by a cutting tool with a single cutting point or corner so mounted and driven that the cutting point will have a reciprocating motion along a line passing through the apex of the pitch cone. If the outline of the large end of the tooth is drawn in its proper position on the gear blank and the cutting point guided along the portion of the curve for one side of the tooth by a feeding mechanism operated while the tool is not cutting, each stroke of the tool will form one element of the tooth surface and, at the end of the operation, one side of a perfect tooth will be completed. By continuing the process for all the teeth one side of all of them will be completed. A similar operation will form the other sides of the teeth.

In the machines which have been designed for the application of this principle to the cutting of bevel gears the tool is mounted on a slide which travels along a swinging arm attached by a universal joint to the frame upon which the gear is mounted, and a point on this arm is guided along a template cut from sheet metal to such a form and so mounted as to guide the tool in the proper manner when the point resting on the template is moved along it by the feed motion. The template is mounted at a greater distance from the apex of the gear than the large end of the gear is. The advantage of a large template is thus gained, which is more easily made for the smaller sizes of teeth, and whose inaccuracies are diminished in the teeth of the gear.

Special forms of bevel-gear teeth are cut by a planing method somewhat similar to the one just given. The essential difference is that the straight side of a planer tool or the flat side of a milling-cutter is used to form the teeth, instead of the cutting point.*

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* Some methods of planing bevel-gear teeth are given in the following articles: Grant's Epicycloid Bevel-gear Generator, *American Machinist*, June 7, 1894; Report of the Committee on Science and the Arts on the Bilgram Bevel-gear Cutter, Journal of the Franklin Institute, August, 1886; "'Planed' Bevel-gear Teeth," *American Machinist*, Dec. 3, 1896. See also *American Machinist*, May 9, 1885.

SKEW BEVEL GEARS.

60. When motion is to be transmitted by a single pair of gears between two shafts which are neither parallel nor intersecting, and it is desired to have line contact, the pitch surfaces of the gears become hyperboloids of revolution generated by a common generatrix which may be considered as the line of contact between the two surfaces. Any corresponding portions of these hyperboloids can be used as pitch surfaces upon which to develop teeth of such a form that they will have line contact. When such teeth work together, they have a sliding motion over each other in the direction of the line of contact as well as the combined sliding and rolling motion which occurs in ordinary spur gearing. This end sliding causes considerable frictional losses and wear when any considerable amount of energy is transmitted, and is a serious objection to their use The teeth are difficult and expensive to make on account of their form. Their application is confined to a few special cases.* The angular velocity ratio of the gears is inversely proportional to the number of teeth, but not to the diameters of the pitch surfaces.

By the use of an intermediate shaft whose axis intersects those of the two given shafts, two pairs of the ordinary bevel gears with teeth whose elements are straight lines passing through the vertex of the pitch cone can be used for the transmission of a positive velocity ratio.

SCREW GEARS.

61. When it is not desired to have line contact between the teeth of the gears on a pair of shafts which are not parallel and do not intersect, gears having helical teeth on cylindrical pitch surfaces can be used. This may be considered as a substitute for skew bevel gears. Theoretically the contact between a pair of teeth is at a point, but practically it extends over a small area whose extent cannot be defined. Such a pair of gears are shown conventionally in

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* Complete discussions of skew gearing are given in "Principles of Mechanism," by S. W. Robinson, 1896, and "Kinematics or Mechanical Movements," by C. W. MacCord, 1889.

Fig. 63. The axes of the gears are $A'A''$ and $B'B''$, and the pitch surfaces are A and B. The point of contact of the pitch cylinders is at P on the common perpendicular to the two axes. Any line drawn through P so as to lie in a plane perpendicular to the normal to the axes and divide the angle $A'PB'$ into two parts, α and β, can be taken as the common tangent to the pitch-surface elements of a pair of teeth in contact at P. α and β may have any value between zero and the angle between the axes.

In gearing of this form the portion of the pitch cylinder which lies near the pitch point and extends on both sides of it must be

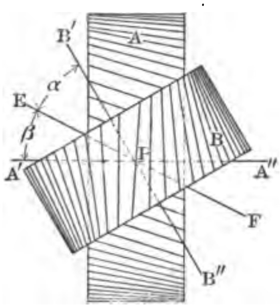

FIG. 63.

used. Screw gearing made in this manner has all the objectionable features of skew bevel gears, together with the additional one of having point contact. The latter is a serious fault when considerable energy is to be transmitted.

The angular velocity ratio is inversely proportional to the number of teeth in the gears.

Worm and Worm-wheel.

62. When two shafts are at right angles and do not intersect, motion can be transmitted between them by a special form of screw gearing, called the **worm and worm-wheel**. In the more common form the worm corresponds to a short length of an ordinary single-thread screw; the screw may have any number of threads, however. Fig. 64 shows conventionally an end view of such a worm in mesh with a worm-wheel which is shown in axial section. The teeth of the worm-wheel embrace a certain angular portion of the

circumference of the worm (usually from 60° to 90°), and their surfaces are swept out by a generatrix having the form of an axial section of the worm-thread. When generating the tooth surfaces the relative motion of the parts must be the same as that of the finished mechanism. The velocity ratio of the worm and the wheel is the same as the ratio of the number of teeth in the wheel to the threads (not turns of thread) on the worm.

If a plane is passed through the axis of the worm, and normal to that of the worm-wheel, the thread sections cut from the worm

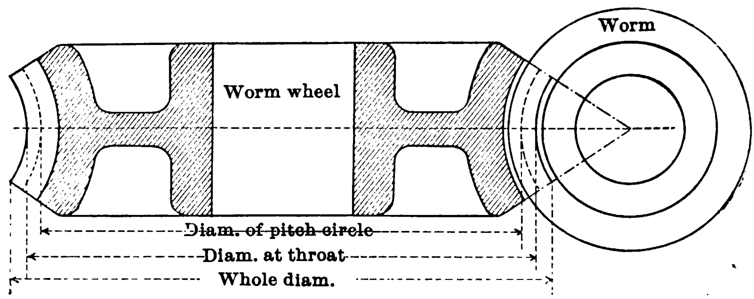

FIG. 64.

correspond to the teeth of a rack, there being several turns of the thread on the worm, and those cut from the worm-wheel are the same as those of a spur-wheel of the same diameter as the worm-wheel on this section. Every other plane parallel to this one will cut the teeth so as to give sections of a different form.*

The cutting of a worm and worm-wheel by machine tools is readily accomplished, especially if the axial section of the worm-thread is bounded by straight lines, the sections corresponding to the involute form of rack tooth. The common practice is to cut a duplicate of the worm in a lathe with a straight-sided and consequently easily made tool. This duplicate is then fluted in the same manner as taps, etc., in order to form cutting edges upon it. It is then made of sufficient hardness to cut the material of the worm-wheel. The piece thus made is called a **hob.**

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* The method of obtaining the sectional forms of the teeth is thoroughly given in "Elements of Machine Design," Part 1, by W. C. Unwin, 1892.

The hob and worm-wheel blank are then mounted with their axes at right angles, but necessarily somewhat further apart than the given distance between the worm and wheel. They are then rotated about their axes with the same velocity ratio as the worm and wheel are to have, and gradually brought nearer together so that the hob cuts away the stock between the teeth until their axes are at the same distance apart as that given for the worm and wheel. By this operation the teeth are given the correct outline.

With teeth so formed, contact between an engaging pair of the worm and wheel at any instant, except when they first touch and separate, is a line of double curvature. Such gears are, for this reason, much better for transmitting energy than the ordinary screw gearing with point contact. It should be noted, however, that the use of such gearing is limited to the single case of the axes at right angles.

It is perfectly practicable, although not quite so satisfactory, to cut the worm-wheel teeth without the aid of a machine for giving the wheel the requisite angular rotation. Such a method is commonly practised as follows: The worm-wheel blank has the teeth first roughed out as nearly as possible to the correct form on an ordinary milling-machine or gear-cutter. It is then mounted on a spindle and left free to rotate. The hob is brought as nearly as possible into the position to be occupied by the worm, the points of its teeth projecting into the roughed-out spaces of the wheel. Upon rotating the hob it drives the wheel, and at the same time removes the stock as they are gradually brought nearer together. When the requisite distance between them is finally reached, the teeth of the wheel have the correct form. Inaccuracies are more apt to occur in this method than when both hob and wheel are driven.

The name "worm and worm-wheel" is also applied to the mechanism in which the teeth of both the worm and wheel are helical. In this form the teeth of the worm-wheel make an angle with its axis which is the same as the thread angle of the worm, measured between a tangent to the thread at the pitch line and a normal to the axis of the worm.

If the axis of the worm deviates from a right angle with the axis of the worm-wheel by an amount equal to the thread angle of the worm, it will engage with an ordinary spur gear.

63. The Hindley worm and worm-wheel are shown conventionally in section in Fig. 65. They are best described by the principles involved in their manufacture.*

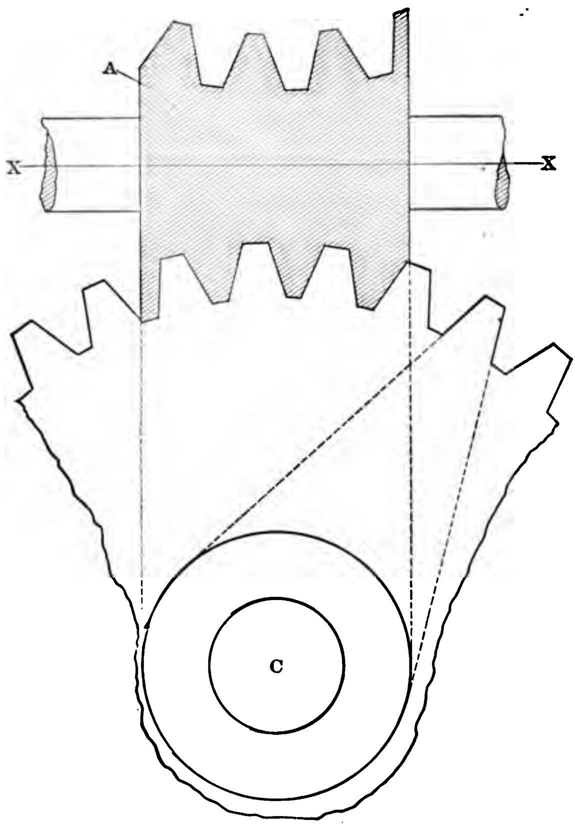


FIG. 65.

A cutting tool with straight sides, and of a form to correspond to the sections of the worm-wheel's teeth cut by a plane perpendicular to its axis and passing through the axis of the worm, is mounted to swing about an axis *C* in the same manner as one of the teeth of the wheel. The worm blank is then mounted so that it is somewhat further from the axis *C* of the cutter than the given

* "The Construction of the Hindley Worm," *American Machinist*, March 25, 1897, etc. A well-illustrated, complete article.

distance between the axes of the worm and wheel. The two are then driven with the given velocity ratio of the mechanism, and the tool cuts a thread in the worm blank. They must, of course, be fed together gradually into the correct position. A hob is then made of the same form as the worm, and is used for cutting the worm-wheel.

The length of the worm is limited to the portion between the two elements where the surface of the thread becomes normal to its axis. In Fig. 65 the sectional form of the teeth of the worm-wheel is such that the extensions of their sides are tangent to the larger circle drawn around the axis C. The length of the worm cannot exceed the diameter of this circle. If made longer, it could not be put into engagement with the wheel after both were made, the hooked form of the thread at the end of the worm preventing such engagement.

The nature of the contact between the engaging parts is difficult to understand on account of the constantly changing form of the worm thread from end to end.

There are two features of this mechanism which are serious obstacles to its use. If the shafts are not at the right distance apart, or if the worm moves longitudinally, the gears are thrown out of proper engagement. Both these faults are apt to come with wear, the axes tending to get farther apart and the worm to get end motion.

While this device has been known for many years, it has not been put to any considerable practical use. Recently, however, it has been applied to elevators built by the Sprague Elevator Co.*

NON-CIRCULAR GEARS.

64. When motion is imparted to one rotating piece by another so that the angular velocity ratio is variable instead of constant, the pitch surfaces must have non-circular forms. Their profiles depend on the nature of the relative angular velocity.

65. Elliptical gears are probably as common as any of the non-circular forms. In a pair the ellipses must be similar and equal. Fig. 66 shows the pitch lines of a pair of such gears. The foci of

* "Electrical Tests of the Efficiency of Worm Gearing," *American Machinist*, Jan. 21, 1897.

Q are A and C; those of R, B and D. They are placed so that the distance $AB = CD = EF$. The latter is the major axis of Q, and is equal to that of R.

When used as gears, each is rotated about one of these foci. Assume that A and B are the centres of rotation. Then the pitch point must lie on the line AB. This is in accordance with the law that the centros of three bodies moving relatively to each other must lie in the same straight line (§ 8). The three bodies in this case

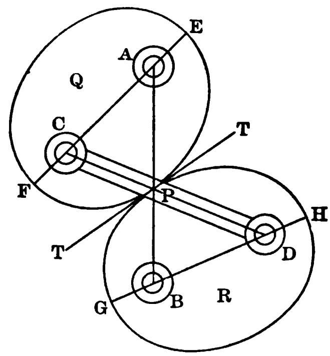


FIG. 66.

are Q, R, and the stationary link articulating with them at A and B.

It can be shown that P, the intersection of AB and CD, is the point of tangency of the ellipses. The proof involves the property of an ellipse, that a tangent to it at any point makes equal angles with the focal lines of that point. In accordance with this, it can be seen that TT drawn through P so as to make equal angles with the focal lines of Q will do the same with those of R. TT is therefore a common tangent to both Q and R at P. P is therefore the pitch point. Since this is true for any position of the ellipses as long as CD remains constant in length, they will roll together in common tangency at a point which moves along AB, if C and D are connected by a link CD, as shown. They can therefore represent the profiles of pitch surfaces upon which teeth can be cut to form gears. Both gears have the same angular velocity when the minor axes of the ellipses meet at the pitch point, forming one straight line. The velocity ratio varies from $AF : GB$ to

AE : *BH*. If in either ellipse lines are drawn from the extremities of the minor axis to one of its foci, they will divide the circumference of a circle into two parts proportional to the angles through which the selected gear will rotate for successive half revolutions on the other gear, starting from the position of tangency at the extremities of the minor axes.

The four links joining the foci form a kinematic chain of four turning pairs with opposite links equal in length. If one link of this chain, as *AC*, were rotated uniformly, *BD* would be driven with the same angular velocity as when the gears are used, due allowance being made for "dead centres."

Gears of this form are used on shaping-machines for a quick return and slow cutting motion. One is driven uniformly, and the other connected to the tool carrying part of the machine. Other motions are more commonly used for this purpose, however.

66. Teeth for non-circular gears can be made according to any of the systems that are suitable for circular gears.

For cycloidal teeth, the describing circle is rolled on the ellipse. With the exception of those at opposite ends of equal diameters, the teeth have different profiles. When the same describing circle is used for all teeth, those at the part of the pitch line having the greatest curvature are weaker than the others. This may be objectionable when the ellipse has considerable eccentricity. It can be overcome by using different rolling circles on different parts of the pitch line. There is no objection to doing this, since the teeth do not have the same form under any condition. The smallest rolling circle should be used at the part of most rapid curvature.

Involute teeth can be generated by using a base cylinder (not circular) after the method for circular gears. The teeth generally have a better form when the base cylinder approaches more closely to the pitch line at the parts of rapid curvature than at the less curved ones. Such a cylinder is obtained by keeping the angle of obliquity constant.

67. Cutting non-circular gears.—Since the tooth spaces of a non-circular gear are all or nearly all different, a large number of cutters would be required for each gear. On account of the expense of making so many, it is customary to rough out the blanks nearly to form on a milling-machine, and finish by hand. A gear cutter

which gives proper spacing and nearly accurate forms to the teeth of elliptical gears is made by George B. Grant.*

68. Gears having a fixed law of motion.—Cases sometimes arise where a pair of gears are to be designed to give a certain motion to the shaft which they rotate, or to drive some part of a mechanism according to a given law. The forms are as numerous and varied as the motions required.† Only one will be given.

69. Pinion and gear for driving a chain belt at a uniform speed.—When a sprocket-wheel, such as is shown in Fig. 67, is rotated

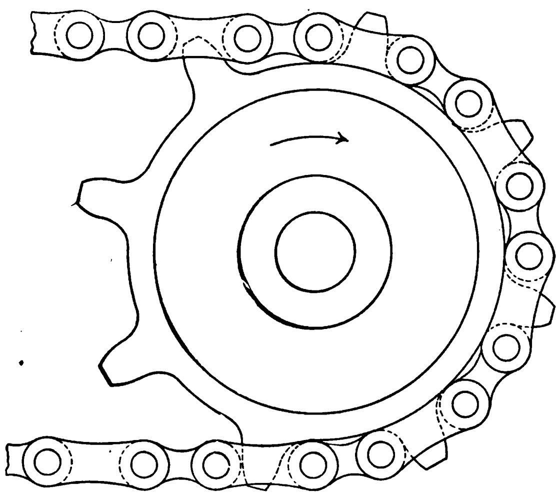


Fig. 67.

uniformly, the part of the chain approaching the wheel is driven at a variable velocity. This is due to the fact that as the chain is wound on the gear it turns successively about each pin as it is brought upon the wheel. The velocity of a pin becomes uniform as soon as the ends of the links which it connects rest upon the wheel. The pin then moves in a circle, and, since the circular motion is uniform, the component parallel to the portion of the chain which is approaching the wheel is variable. The variation is slight, how-

* "A Treatise on Gear Wheels," by George B. Grant, 1893.

† A very full treatment of irregular gears is given in "Principles of Mechanisms," by S. W. Robinson, 1896.

ever, when the number of links required to encompass the sprocket-wheel is as great as shown in the figure.

When a sprocket-wheel having only four teeth, as in Fig. 68, is used, the variation is excessive when the rotation is uniform. This can be seen by inspection. If the wheel rotates so as to wind the

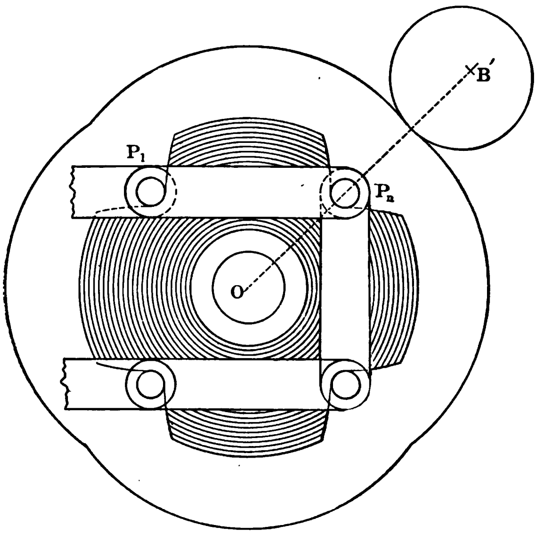


Fig. 68.

chain on the upper side, the pin P_1, which has just come to rest on the sprocket, will travel with uniform velocity in the arc of a circle of radius OP_1 until it leaves the wheel. While the pin moves from P_1 to P_n, the link just winding on the wheel turns about the pin as an axis and has a horizontal linear velocity equal to the horizontal component of the circular motion of the axis of the pin. In order for the free portion of the chain, which in this case is horizontal, to have a uniform linear velocity, the sprocket-wheel must be driven at such a variable angular velocity as will make the horizontal component of the pin's velocity constant.

If power is furnished by a uniformly rotating shaft, as is commonly the case, the sprocket-wheel shaft can be driven by a pair of non-circular gears. Thus, if B', Fig. 68, is the centre of the uniformly rotating shaft which is to make one revolution for each link wound on the wheel, the pitch line of its gear takes the single-lobed

form shown. The gear connected to the sprocket-wheel and revolving about the same axis with it, has four similar and equal lobes as shown. The problem thus resolves itself into finding the pitch lines of a pair of gears such that while the driver rotates uniformly, the driven one will have such an angular velocity as will give the chain a uniform linear velocity.

For convenience the driving-wheel will be called the pinion.

In solving the problem the following notation is used:

R = pitch radius of sprocket-wheel, i.e., the radius of the circular arc in which the axis of a chain-pin travels while the pin rests on the wheel.

d = distance between gear centres.

l = length of links between centres of pins.

n = number of sides on polygon forming the outline of sprocket-wheel.

v = uniform linear velocity of chain.

θ = angle of sprocket-wheel embraced by one link of the chain.

ϕ = angle, at any instant, between the centre line of the approaching side of the chain and the radial line joining the centre of the sprocket-wheel to the centre of the last pin wound on it. The limiting values of ϕ are

$$90^\circ - \frac{\theta}{2} \quad \text{and} \quad 90^\circ + \frac{\theta}{2}.$$

ω = angular velocity of sprocket-wheel at any instant.

In Fig. 69, O is the axis of the sprocket-wheel and its gear, and B' that of the pinion. CL is drawn parallel to the approaching side of the chain, which is considered as having a constant direction. P_1P_n, parallel to CL, is the position of a centre line of a link that has just been wound on the wheel, P_1 being the first position in which the pin at the end of the link articulating with the approaching side of the chain becomes the axis about which the chain bends; the same pin remains the axis during its motion from P_1 to P_n.

For convenience, assume that the mean angular velocity of the sprocket wheel $= \omega =$ one revolution per minute. Such an assumption does not affect the design in any way. Then

$$v = nl.$$

Let P be any position of the centre of the pin during its motion from P_1 to P_n. First, it is necessary to find the linear velocity at which P must move in order to give the chain its velocity v. To do this take PF parallel to CL and of such a length as to represent v according to any convenient scale; draw FH perpendicular to PF, and PH tangent to the pitch circle CPL of the sprocket-wheel;

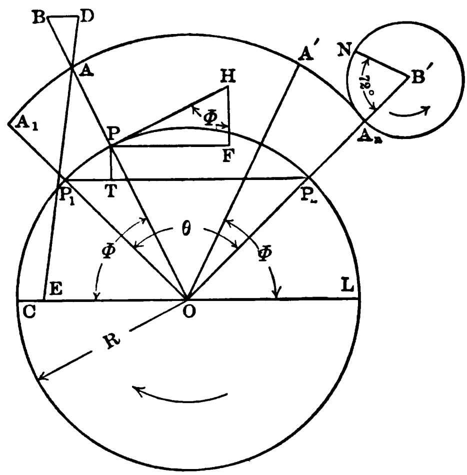


FIG. 69.

then $PH = v \div \sin \phi$ is the linear velocity at which P must move while in the position shown.

The angular velocity of the sprocket-wheel for this position of P, is found by the equation

$$\omega = \frac{PH}{2\pi R} = \frac{v}{\sin \phi} \cdot \frac{1}{2\pi R}.$$

By taking a single-lobed pinion which makes one revolution for each link wound on the sprocket-wheel, as has been assumed, it must make n revolutions for one of the wheel, and since the latter has been assumed to make one revolution per minute, the pinion must make n revolutions per minute. In this particular case $n = 4$.

It now remains to find the pitch radii that the gears working together must have for the assumed position of P, in order to give

the sprocket-wheel an angular velocity ω, as found by the preceding equation, while the pinion is rotating at its uniform velocity of n revolutions per minute.

Draw the radial line OP and produce it, making $OB = OB'$; take OE, in any convenient direction, to represent the revolutions of the pinion per minute $= n$, and draw BD parallel to OE to represent the angular velocity of the sprocket-wheel $= \omega$, in accordance with the same scale; draw ED intersecting BO at A; then OA is the required radius of the sprocket gear for the assumed position of P, which must be measured as the distance OA' on a line making the angle ϕ with CL, A' being the point which will lie on OB' when the centre of the chain-pin is at P. A' is therefore a point in the pitch curve of the gear. But the nature of the problem calls for a lobe symmetrical about a line bisecting the angle θ; therefore A is also a point in the pitch curve.

The point of the pitch curve of the pinion-wheel which will coincide with A' when the chain-pin is at P, can be found as follows: Draw PT perpendicular to P_1P_n, and find the ratio $P_1T \div P_1P_n$. For convenience it will be assumed to be 0.2 in the present case. This indicates that 0.2 of the length of one link of the chain has been drawn in, and since the pinion rotates uniformly at the rate of one revolution for each link drawn in, the pinion must have made 0.2 of a revolution $= 0.2 \times 360° = 72°$, while moving P through the arc P_1P.

With B' as a centre, take the angle $OB'N = 72°$, and make $B'N = AB$ found by the construction for the radius OA; N is then one point of the pinion pitch curve.

By taking other positions of P, enough points of the curve A_1A_n can be found to determine its form, and at the same time the points necessary to determine the pitch curve of the pinion, which is symmetrical about OB'.

There will, of course, be n lobes on the sprocket-wheel gear, all having the same form as the one just determined, their ends joining each other.

If the centre of the pinion were moved around O through any angle β (not shown), and used to drive the mechanism while in that position, the sprocket-wheel gear would have to be rotated through an equal angle relatively to the sprocket-wheel in order to keep the

motion of the chain uniform, due allowance being made for the fact that no adjustment of the gear-wheel is necessary when the angle through which the pinion shaft is moved is the same as that embraced by one or more links of the chain, and that the angle β may be increased or decreased by one or more of the angles embraced by a link, in determining the amount of rotative adjustment necessary for the gear.

If a driving pinion having two lobes is used, the line OE becomes only one half as long by the same scale, and the angle $OB'N$ half as great as for a single lobe. The same principle holds good for any number of lobes on the pinion.

70. Non-circular bevel and screw gearing can also be made with due regard to the principles for determining the pitch surfaces that have been given in the preceding paragraph. The difficulties of manufacture become so great, however, that they are very seldom made except as examples of the possibilities of such construction.

TRAINS OF GEARING.

71. In any train of spur gears such as is shown in Fig. 70, where the gears are represented by their pitch-surface profiles, and concentric ones are assumed as rigidly fastened together, if the

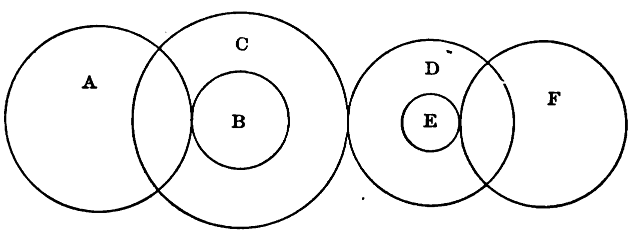


FIG. 70.

angular velocity of the first is given, that of the last can be found by the following equation, in which the letters indicating the gears may also be taken as representing their respective diameters or radii, as may be the more convenient. $V°$ indicates the angular velocity of the gear represented by the symbol immediately following it.

$$V°F = (V°A)\frac{A}{B} \cdot \frac{C}{D} \cdot \frac{E}{F}$$

By this it can be seen that in any train of gearing of the form of Fig. 70, if the first is the driver, the angular velocity of the last is equal to that of the first multiplied by the radii of all the succeeding drivers, and divided by the radii of all the driven ones. This is true for any number of gears. It also applies to annular as well as spur gears. In the latter the direction of rotation of the engaging gears is the same.

If any gear of a train is both the driver and the driven, the velocity ratio of the two gears with which it meshes is the same as if

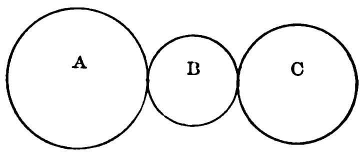


Fig. 71.

they were in direct engagement; the direction of rotation is changed, however. Thus, in Fig. 71, A and C both rotate in the same direction with the same angular velocity ratio as if meshed together; when meshing together, they rotate in opposite directions. The intermediate gear B is called an **idler**.

The principles just cited also apply to bevel gearing with intersecting axes. The equation for relative angular velocity can be applied by using the largest diameters of the bevel gears.

CHAPTER III.

COUPLINGS.

72. The Universal Joint or Hooke's Coupling is sometimes used to connect intersecting shafts when the angle between them is not too great (45° or less). Its general form is shown in Fig. 72. A and B are two shafts intersecting at O, the angle between them being θ. To the end of each shaft a pair of forked arms are attached, which are joined together by a link with two double arms

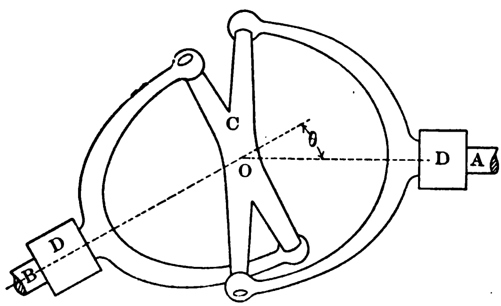


FIG. 72.

at right angles to each other and intersecting at O. These arms are generally of equal length.

There is an objectionable property belonging to this coupling, which is that the velocity ratio of the two shafts is a constantly varying one, although they make *quarter* revolutions in equal times. The change of velocity ratio varies with the angle between the shafts, increasing as it increases.

The proof that equal angles are not passed through in equal times can be given with the aid of Figs. 73 and 74. Fig. 73 is the projection of the joint on a plane parallel to the plane of the shafts A and B. Fig. 74 is a line projection on a plane perpendicular to the shaft A, the arms of the cross being represented by the lines

PQ and *RS*. Every point in the axis of the arm *HK*, Fig. 73, travels in a circle whose plane is perpendicular to *A*, the projection of the plane being *HK*. Also, every point in the axis of

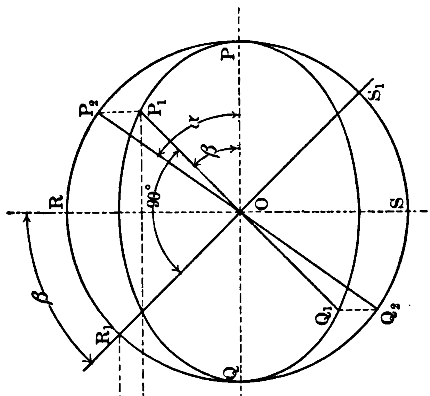


Fig. 74.

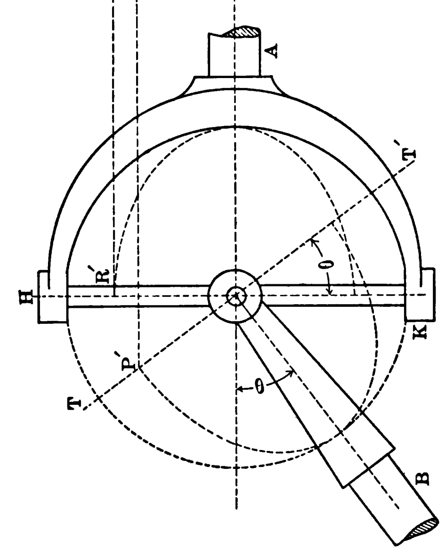


Fig. 73.

the arms of the cross attached to the fork of *B* travels in the same manner relatively to *B*, the projection of the plane of the circle being *TT'*. The path of a point in the arm connected to *B* projects as an ellipse in Fig. 74. The major axis of the ellipse is the diameter

of the circle in which the point travels, and the minor axis is the diameter multiplied by cos θ.

When A has rotated through any angle β, so that RS takes the position R_1S_1, the projection of PQ will be P_1Q_1, *perpendicular* to R_1S_1. This is the projection of the arm as it lies in a plane making an angle θ with the plane of the paper, and therefore, in order to find the real angle through which B has rotated, it is necessary to rotate the plane about QP until it coincides with the paper. (This is practically placing the two shafts in line by bringing B perpendicular to the paper in Fig. 74 without allowing it to rotate about its axis. This would require loosening one of the forks from the cross.) P_1 would fall at P_2, and the real angle through which B has been driven by A is α, in this case larger than β. When a quarter revolution has been passed through by A, it is evident from the figure that the same angle will have been passed through by B. In the second quadrant, B will at first fall behind A, regaining its original relative angular position again at the end of the quadrant. The conditions of the third and fourth quadrants are the same as those of the first and second.

Two shafts A and C, Fig. 75, lying in the same plane and making equal angles with an intermediate shaft B, will have a con-

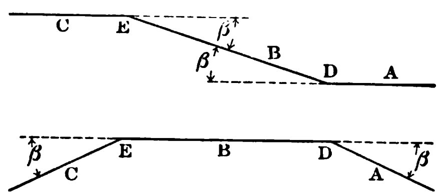


FIG. 75.

stant angular velocity ratio when connected by Hooke's couplings at D and E, provided the two forks on B lie in the same plane. By this arrangement the irregularities due to one coupling are eliminated by those of the other. The value of β does not affect the accuracy of working, so that two parallel shafts, as shown in the upper part of Fig. 75, may be moved relatively to each other and will work correctly as long as they are kept parallel.*

* See feeding mechanism on Brown and Sharp's milling-machines and driving device on adjustable multi-spindle drilling-machines.

It is plain that the intermediate shaft B may have any length. When it is made very short, the whole mechanism becomes a *double universal coupling.*

When the three shafts A, B, and C do not lie in the same plane, A and C can be given the same angular velocity by placing the forks on the ends of the intermediate shaft B so that when one lies in the plane of A and B, the other will lie in the plane of B and C.*

73. Almond's angular coupling.—Fig. 76 represents a coupling for intersecting shafts making any angle within practical limits, which gives the same angular velocity to both at any instant. The frame, bearings, and rod F form a single rigid piece. A cap or disk m is attached to the end of the shaft A. A pin p passes through the cap, perpendicular to and intersecting the axis of the shaft. This pin carries an arm C, which turns about p as a pivot, the opposite end having a ball-and-socket bearing into which fits the end of an arm a that is part of the casting *aeb*. When A rotates, the arm c is driven by the pin p, which in turn drives a. The casting *aeb* is thus given a motion of rotation and translation upon the rod F. The remaining parts of the coupling are exact duplicates of those described, and the arm b is a duplicate of a, so that the shaft B is driven by b acting upon d.

In Fig. 77 the centre lines of the coupling are shown in two positions on the coordinate planes H, V, and S. The first position is the same as that of Fig. 76; the second is that after the shaft has rotated through an angle α, the rocker-arm on F moving through an angle β at the same time, and the shaft B through an angle α'. The notation is the same as that of Fig. 76, with a few additions to be explained later. The letters are primed to indicate the position after movement has occurred.

The point t, which is the intersection of a and c, is constrained by the arm a to move in the surface of a cylinder whose centre is the axis of F, and whose radius is a; t is also constrained by c so that it must always remain in the surface of a sphere of radius c, whose centre is at the intersection of c and A. Since t must lie in the surfaces of both a cylinder and a sphere, its path (locus) is their

* For further discussion of the Hooke's coupling, see "Principles of Mechanisms," by R. Willis, 1870.

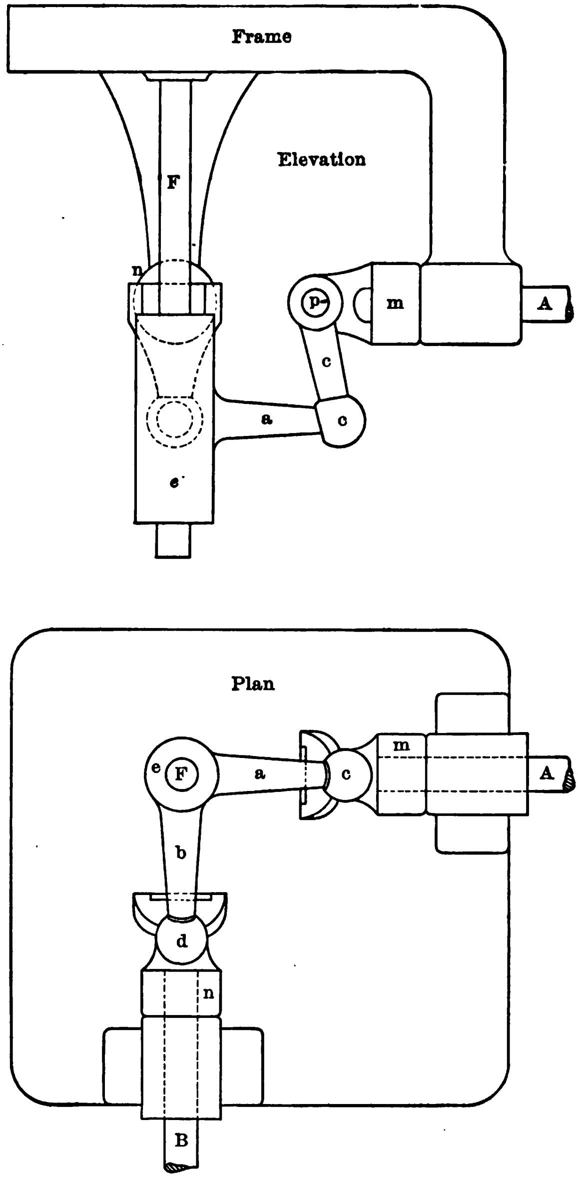


FIG. 76.

intersection. A portion of this path, gh, is shown on (S); the projection on (H) is an arc of a circle with centre at F and radius a. The path of r, the intersection of b and d, is exactly the same as

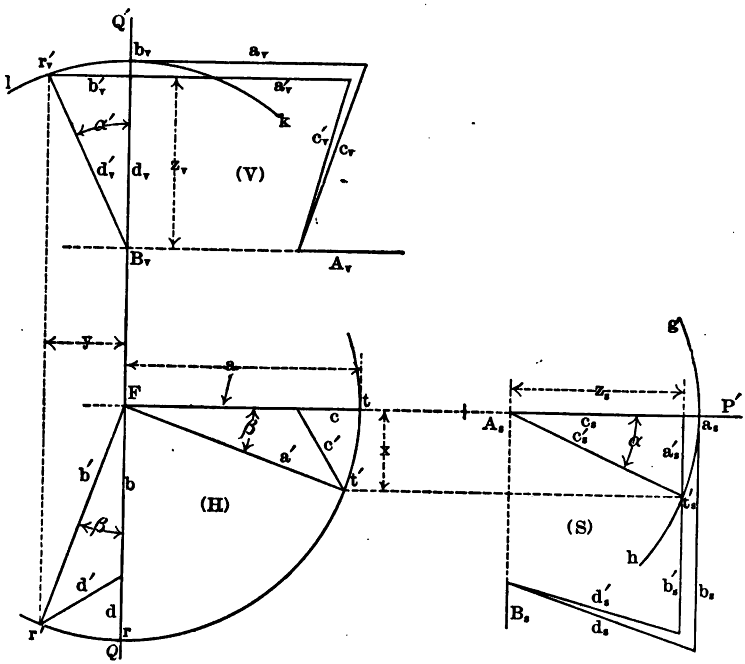


FIG. 77.

that of t in form and extent, for the operation of the mechanism requires b and d to be of the same length as a and c respectively, and to occupy the same relative position with regard to F. A portion of this path is shown in the curve kl on (V).

Suppose, now, that A has rotated through an angle α and the angle of B's rotation is required. A is projected on (S), Fig. 77, as a point A_s. The arm c, attached to A, makes the angle α with its initial position, and is now projected at c_s'; its articulation with a must lie on gh and is, therefore, at t_s'. The horizontal projection t' is at the intersection of a line through t_s', perpendicular to QQ', with the circle of radius a and centre F; QQ' is parallel to the ground line of (H) and (S); joining F and t' gives the angle β,

through which a has moved, and the position, a', after the motion. Since a and b are rigidly connected (practically they are one piece) they must both rotate through equal angles, so by drawing b' at an angle β with its first position, its intersection with the same circle upon which t' lies, gives r', which is the articulation of b and d; r_v' is found by drawing a perpendicular to the ground line of (H) and (V), intersecting kl at r_v'; joining r_v' and B_v (the latter is a point) gives the position d'_v, of d after rotation, and also the angle α'.

Now pass two planes through F perpendicular to (V) and (S) so that their traces are FP' and FQ'. The distance of both t_s' and t' from FP' is x; the distance of r' and r_v' from QQ' is y. But $a' = b'$, and they make equal angles with PP' and QQ' respecttively, therefore $y = x$, i.e., t_s' and r_v' are at equal distances from the planes in which they were lying in their initial positions, and, since they travel in similar paths and are restrained by links of equal lengths, the angles of similar links with their respective planes must be equal; therefore $\alpha' = \alpha$; but α' measures the angle of B's rotation, therefore both shafts have passed through, equal angles, and since α is any angle, their velocity ratio is constant and equal to unity.

The same general form of coupling can be used for a constant velocity ratio even when the shafts do not intersect, including parallel shafts. In such cases the axis of the standard, F, must intersect the axes of both shafts at right angles, and the arms a and b must lie at a distance apart, measured along F, equal to the length of the part of F lying between its intersections with the two shafts.

The angle between the rigidly connected arms a and b must be equal to that between the shafts connected by the coupling. When the axes of the shafts are coincident, they will rotate in opposite directions. Parallel shafts on the same side of F will rotate in the same direction; those on opposite sides of F, in opposite directions.

Another form of the Almond coupling has duplicate rigid crank-arms attached to the ends of the shafts. Each arm has a ball at its effective radial length. The ball has a hole through it to receive one of the arms a and b. These are turned to a uniform diameter so as to slide freely through the balls.

Still another form has the same rigid cranks on the shafts, but

instead of having the arms *a* and *b* made to slide through the ball, they are made telescopic, one end being a part of the spherical portion of the universal joint.

74. The Oldham coupling, Fig. 78, can be used for connecting shafts which are slightly out of line but whose axes are parallel or

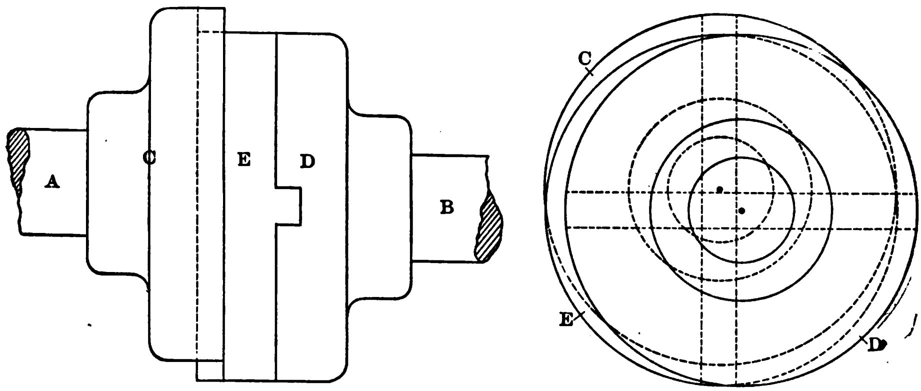


FIG. 78.

very nearly so. Duplicate flanges, *C* and *D*, are attached to the end of each shaft. Each flange has a groove running across the centre of its face. Between these flanges is a disk *E*, with two tongues running across its opposite faces at right angles, which fit into the grooves of the flanges as shown. Since the grooves of the flanges are always held at right angles to each other by the tongues on the intermediate disk, both shafts must always turn with the same angular velocity. The sliding of the tongues in the grooves allows for the lack of alignment of the shafts.

This coupling is used for the main shafts of steamships, dynamos, engines, etc., where the shafts are liable to be thrown slightly out of alignment by slight movements of the bearings.

CHAPTER IV.

BELT GEARING.

75. There are many cases arising in practice where one shaft or rotating piece is to act as a driver for a second one, it being necessary to have only an approximately constant velocity ratio between them. Belting or bands are used for this purpose with entire satisfaction. By far the most common is the flat band running on circular cylinders or pulleys attached to and concentric with the shafting. Round, square, triangular, and other forms are more limited to special requirements.

If a material perfectly inextensible could be used for a belt, and run without slipping on the pulleys, a constant velocity ratio would be attained. While this is by no means the condition in practice, it is customary to assume it as true for convenience in calculating the diameters of pulleys and their angular velocities, and then to allow for the discrepancy by an amount which practice shows to be correct under the existing conditions. The kinematics of belting will, therefore, be dealt with as if the belt were inextensible and without slip on the pulleys. The thickness of the belt is also neglected.

76. Parallel shafts and guidance of belts.—Fig. 79 shows an *open* belt connecting pulleys on the parallel shafts A and B. Every part of the belt moves with the same velocity (according to the assumptions of the preceding section), and, since the belt is in contact with both pulleys, the linear velocities of their surfaces are equal. Their angular velocities are, therefore, inversely proportional to their radii. The shafts connected with the open belt rotate in the same sense. In order to have them rotate in opposite senses, the belt must be *crossed* as shown in Fig. 80.

When the velocity ratio of two shafts is very large, it becomes inadmissible, on account of practical reasons, to connect their pul-

leys directly by a belt. In such a case an auxiliary shaft with two pulleys of different diameters is used. This additional shaft is

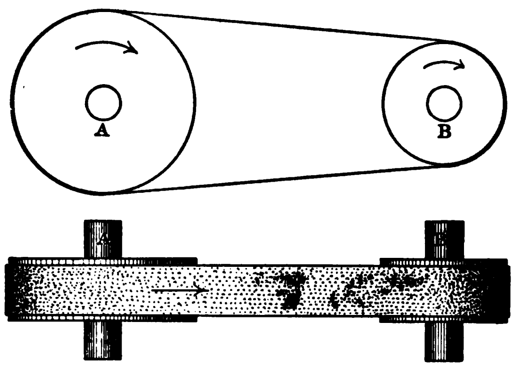


FIG. 79.

generally called a *counter-shaft* or *jack-shaft.* Its velocity is intermediate between those of the driving and driven shafts. The velocity ratio of the first and last pulleys of the set is found in the same way as for a train of gears (§ 71).

For example, suppose that a circular saw is to be driven at 3000

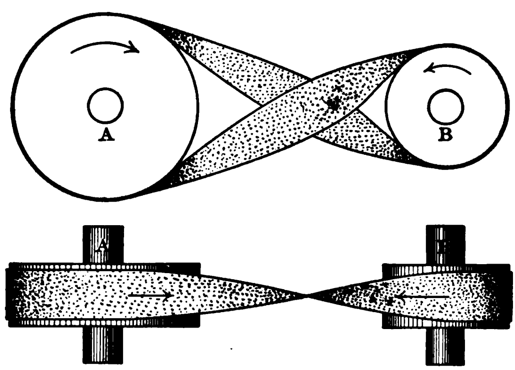


FIG. 80.

revolutions per minute, power being supplied by a line shaft making 150 rev. per min. The velocity ratio is 1 to 20. This would give $1 : \sqrt{20} = 1 : 4.47 +$ for equal ratios of line shaft to counter-

shaft, and counter-shaft to saw. (See Fig. 81.) But there is no reason why these ratios should be equal, while, for practical reasons in making pulleys, etc., it is customary to use values differing from these.

Choose the ratio of the line shaft to the counter-shaft as 1 : 4, then that of the counter-shaft to the saw must be 1 : 5. Suppose that the largest pulley that can be used on the line shaft is 32″ in

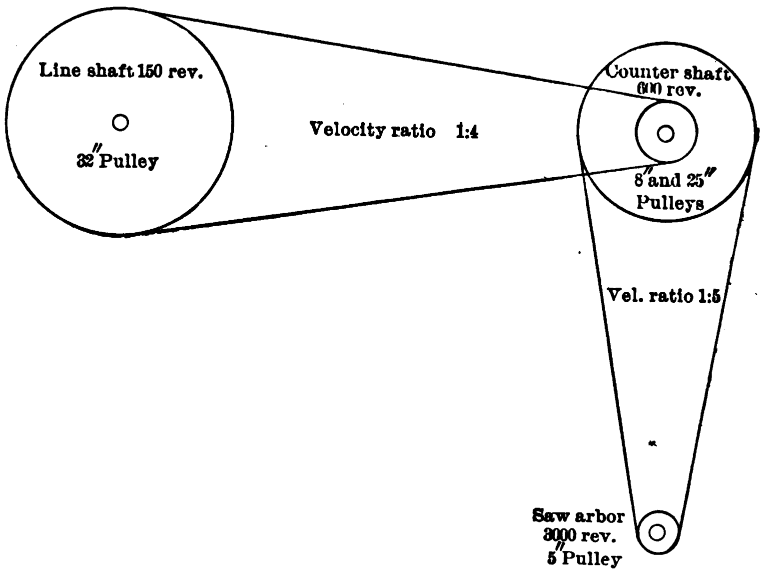


FIG. 81.

diameter, then the one which carries the same belt on the counter-shaft must be $32'' \div 4 = 8''$ in diameter; while the other one on the counter-shaft, carrying the belt leading to the saw, must be $5 \times 5'' = 25''$.

If a side pressure is applied to the edge of a belt that is approaching a cylindrical pulley, as the pressure P, in Fig. 82, the belt will be thrown out of its position normal to the axis of the pulley, and will be shifted along the pulley from end to end in the direction of the force. Hence it is evident that **in order for a belt to maintain its position on a pulley, its centre line on the approaching side must lie in a plane perpendicular to the axis of the pulley.**

As a belt is never actually uniform and straight, and is liable to run off the pulley, especially if slipping occurs, the faces of pulleys

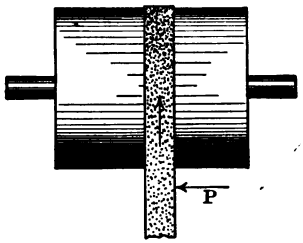


FIG. 82.

whose belts are not to be shifted, are turned crowning (convex), as shown in Fig. 83. When the belt is on either side of the pulley, the

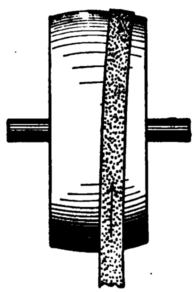


FIG. 83.

crowning face throws it into the position shown in the figure, thus causing it to move to the centre of the pulley, where it will remain as long as the normal conditions of running exist.

77. Shafts neither parallel nor intersecting.—Pulleys on shafts having this relation can be placed so that a plane through the centre of either pulley perpendicular to its shaft, will pass through the centre of the rim of the other pulley at the point of tangency of a line lying in the plane and tangent to both pulleys. This fulfils the conditions, stated in the preceding paragraph, for a belt to maintain its position on a pulley.

Fig. 84 shows the position of the pulleys on two shafts making an angle of 90°. The belt will run only in the direction indicated

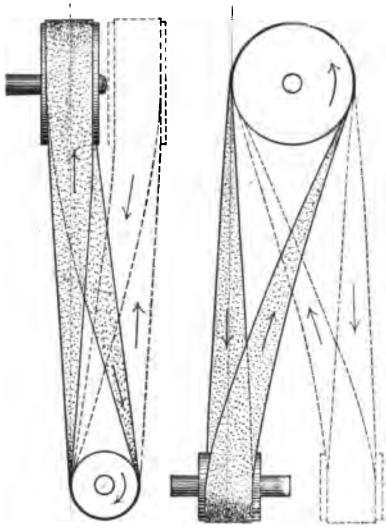


FIG. 84.

by the arrows. In order to have rotation in the opposite direction, each pulley would have to be moved along its shaft a distance equal to the diameter of the other, so that they would occupy the positions indicated by the broken lines.

Two shafts A and B, Fig. 85 (perpendicular to the paper), can be connected by belting running on pulleys properly placed when

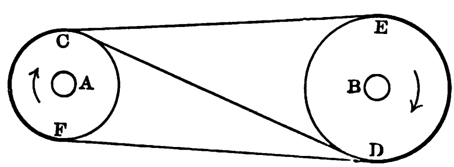


FIG. 85.

the shafts are parallel, and then they may be thrown out of parallelism by rotating either shaft about a line CD, drawn tangent to the pulleys at their face centres on the sides where the belt leaves them. For suppose that the belt and line CD are tangent to the pulleys at

the same points C and D (which is practically true when the distance between the shafts is several times the diameter of the larger pulley), then, if B is rotated about CD, the portion CE of the belt, which is advancing toward B, will always remain in the plane passing through E, D, and C, which is the plane through both the centre of the pulley on B and the point where the centre of the belt leaves the pulley on A, and therefore the belt will always be guided correctly toward B; the point D does not move during the rotation, hence the portion of the belt approaching the pulley on A will lie in the plane through the centre of the pulley. The belt, therefore, will remain in the correct position upon the pulleys.

78. Pulleys for intersecting shafts.—When two shafts lying in the same plane are not parallel, but lie as shown in Fig. 86, it is not

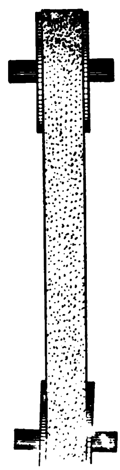


FIG. 86.

possible to place pulleys on them in such a way as to make a belt run upon them. A glance at the figure will show that the pulleys can not be placed so that the belt will be guided properly toward both of them. A pair of auxiliary guide pulleys, called idle pulleys, can be used, however, as shown in Fig. 87, where A and B are the two shafts intersecting at C; the idle pulleys are placed at D, so as

to be tangent to the planes passed through the centres of the pulleys on A and B. The idlers may both be on the same shaft or on separate ones, which may be parallel or not. The diameters of the

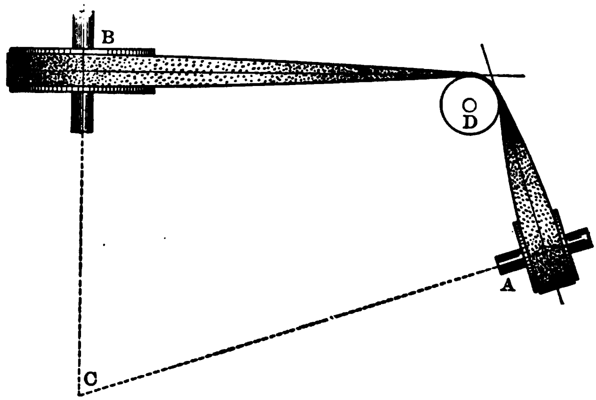


FIG. 87.

idlers are dependent upon practical conditions only. An infinite number of positions of the pulleys on A and B will permit proper guidance of the belt.

79. Drum for shifting a belt.—As explained in § 76, a running belt can be moved along a pulley by pressure applied against one of its edges. This method is commonly practised when belts are to be shifted from one pulley to another, as on counter-pulleys for driving lathes and other machinery which is frequently stopped and started. It is perfectly satisfactory for narrow belts.

When a very wide belt is to be shifted, however, the necessary pressure against its side becomes so great as to injure it by bending up the edges and making them irregular. Some other means must, therefore, be adopted in order to obtain satisfactory results.

The method illustrated in Fig. 88 has been found very satisfactory. The shafts AA and BB are parallel; the axis CC, of the drum E, lies in a plane parallel to that of the pulley-shafts, but makes an angle with the shafts. When the belt is run in the position shown, E is kept free from it. In order to shift the belt so that it will run on D, the drum E is pressed against the belt, and, on

account of its angularity, gradually guides it over to D. The drum must then be removed from contact with the belt. It is assumed that AA is the driver.

It is found in practice that an angle of 75° between the axis of the drum and the centre line of the belt gives good results.

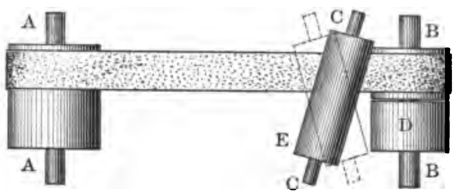


FIG. 88.

For shifting the belt from D back to the original position shown, the drum can be swung around so as to make the same angle with the belt in the opposite direction, as shown by the dotted lines, and pressed against it, or another drum can be placed on the opposite side of the same clear stretch of the belt, in the position of the dotted lines.

80. Length of belts.—In Fig. 89 the shafts A and B are con-

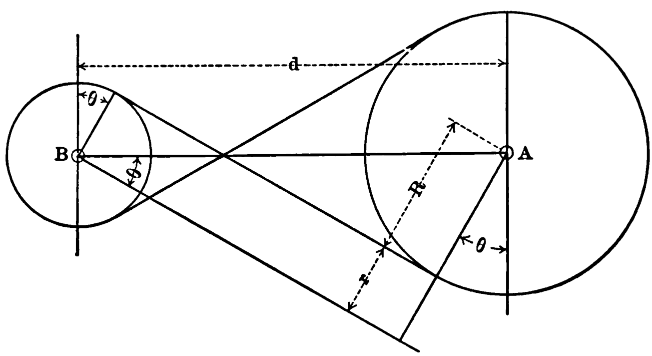


FIG. 89.

nected by a crossed belt. The quantities given are:

d = distance between shafts;
R = radius of large pulley;
r = radius of small pulley;
θ = angle of belt with centre line AB.

The length L of the crossed belt is found by the formula

$$L = 2\sqrt{d^2 - (R + r)^2} + (R + r)\left[\pi + 2 \text{ arc sin}^{-1} \frac{R + r}{d}\right]. \qquad (1)$$

The formula for an open belt, shown in Fig. 90, when the same notation is used, is

$$L = 2\sqrt{d^2 - (R - r)^2} + \pi(R + r) + 2(R - r)\left[\text{arc sin}^{-1} \frac{R - r}{d}\right]. \quad (2)$$

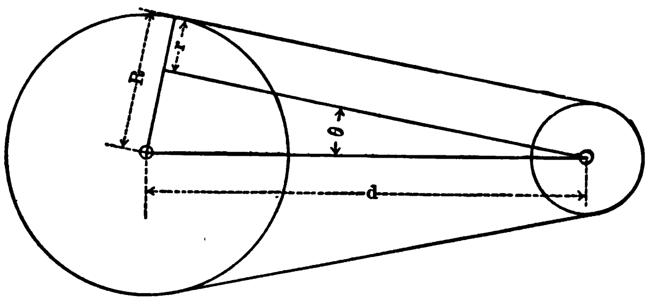


FIG. 90.

81. Stepped or cone pulleys.—In such machine tools as lathes, drill-presses, milling-machines, boring-mills, etc., it is desirable to have a variation of speed. In order to obtain this variation when power is applied through belting, several pulleys of different diameters are placed side by side on the driving-shaft, and a corresponding set runs on the driven shaft which is a part of the machine. All of the pulleys of a set on one shaft are generally cast in one piece, which is called a stepped or cone pulley. These pulleys must be placed so that when the belt runs on the smallest step of either one, it will run on the largest step of the other. In other words, the cone pulleys are placed with their large ends in opposite directions.

Since the same belt is used on all the steps of the pulleys, it is necessary not only that a pair of mates lying in the same plane shall give the correct velocity ratio, but they must also be of such diameters that the same length of belt is required as is suitable for all other pairs.

When the belt is crossed, the length is obtained from formula (1), § 80, in which the only variables are R and r. But their sum is always dealt with as a quantity; therefore, in order to give a constant value to L, it is only necessary that $(R+r)$ shall be a constant, i.e., the sum of the radii of any pair of steps on cone pulleys carrying a crossed belt must be constant.

If the radii R' and r' of a pair of steps are given, the radii R and r of another pair having the ratio $R \div r = a$, can be found by the following formulæ:

$$R \div r = a; \qquad \text{therefore} \qquad R = ar,$$

and $$R + r = R' + r', \quad \text{or} \quad ar + r = R' + r'.$$

Therefore $$r = \frac{R' + r'}{a+1}, \qquad \text{and} \qquad R = a\frac{R' + r'}{a+1}.$$

The open belt does not have a formula for obtaining its length that admits of such easy application for determining the diameters of the steps of the pulleys. In this case both the sum and difference of the radii enter. (See equation (2), § 80.) If R and r vary in value, the term $(R-r)$ would vary, causing two of the terms of

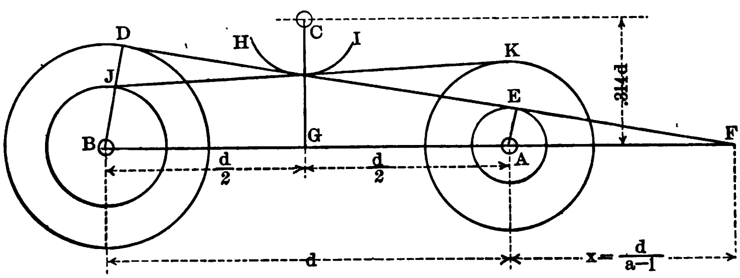


FIG. 91.

the equation to vary, thus making a solution of the equation for constant values of L practically impossible.

The following approximate graphical method is applicable:* Having given the distance d between the shafts A and B, Fig. 91,

* "An Improved Method for Finding the Diameters of Cone and Step Pulleys," by C. A. Smith in Trans. Amer. Soc. Mech. Eng., vol. x. p. 269.

and the radii of one pair of steps, describe the circles to represent them and draw a straight line tangent to them at J and K; at G, the centre of AB, erect a perpendicular GC, of a length equal to $0.314d$ (the coefficient 0.314 is a quantity experimentally determined), and with C as a centre describe the arc HI tangent to JK. Now suppose that the radius BD of one step of another pair is given; the radius AE of the corresponding step is found by describing the circle of radius BD and centre at B, and drawing a tangent DE to it and the arc HI. A circle with A as a centre, drawn tangent to DE, has a radius such that the same length of belt will run on the second pair of steps as on the given pair.

The solution can also be made when one pair of steps and the velocity ratio a of another pair are given. In Fig. 91, extend DE until it intersects the line of centres at F, whose distance from A equals x. Then, from similar triangles, and by taking a as the angular velocity ratio of A to $B = BD \div AE$,

$$\frac{x+d}{x} = \frac{BD}{AE} = a; \quad \text{therefore} \quad x = \frac{d}{a-1}.$$

From this it can be seen that by taking FA equal to $d \div (a-1)$, and drawing a line through F tangent to HI, the circles with centres at A and B, tangent to FD, will give the required velocity ratio and take the proper length of belt.

When the smaller pulley is at B, the value of a becomes less than 1. The value of x thus becomes negative, which indicates that it must be measured from A toward the left in the figure.

For extreme velocity ratios the length of the perpendicular at the middle of the line of centres is slightly different from the value given ($0.314d$), but the difference is so slight as not to need consideration except in cases where great accuracy is desired.*

82. Geometrical series of cone-pulley speeds. — When one stepped pulley runs at a constant speed and that of the other is changed by shifting the belt from one pair of mates to another, as for a lathe, it may be desirable to have the changes of speed bear a

* For the other values see original article in Trans. Amer. Soc. of Mech. Engs. When the angle between the belt and centre line of the pulleys exceeds 18°, the distance CG is taken as $.298d$.

constant ratio to each other, forming a geometrical series. Thus, for a five-stepped cone the desired speeds may be 20, 40, 80, 160, and 320 revolutions per minute, the ratio for each change being 2.

If the highest and lowest speeds are given, together with the number of steps, it becomes necessary to find the geometrical ratio. This can be done by the ordinary method, of course. For convenient reference it will be given here.

For the general case take

N = revolutions per minute when the belt is on the smallest step of the driven cone;

n = revolutions per minute when the belt is on the largest step of the driven cone;

S = number of steps;

G = geometrical ratio of successive speeds.

Then $$G = \left(\frac{N}{n}\right)^{\frac{1}{S-1}}.$$

From this value of G the speeds of the intermediate steps can be determined readily.

As a numerical example, take the following:

$$N=900; \quad n=50; \quad S=6.$$

Then $$G = \sqrt[5]{\tfrac{900}{50}} = \sqrt[5]{18} = 1.782.$$

And the revolutions throughout the range, given in the nearest whole number, are

50, 89, 159, 283, 505, 900.

The remainder of the solution is the same as given in § 81.*

A method of locating the points corresponding to F of Fig. 91, and of obtaining a geometrical velocity ratio of speeds, both graphically, is shown in Fig. 92, where AB is the distance between cone centres, and PD and Hd pass through A and B respectively, and are perpendicular to AB.

* Professor Robinson, in his "Principles of Mechanisms," gives a complete graphical solution for a geometrical series of speeds. It is given briefly here, by his consent.

If the constant speed of the counter-shaft and one speed of the lathe-spindle, which may be the fastest, are given, the solution for finding the diameters of the cone-steps for this speed is as follows:

Take AD, according to any convenient scale, as the speed of the counter-shaft, and Bd, to the same scale, as the speed of the lathe-

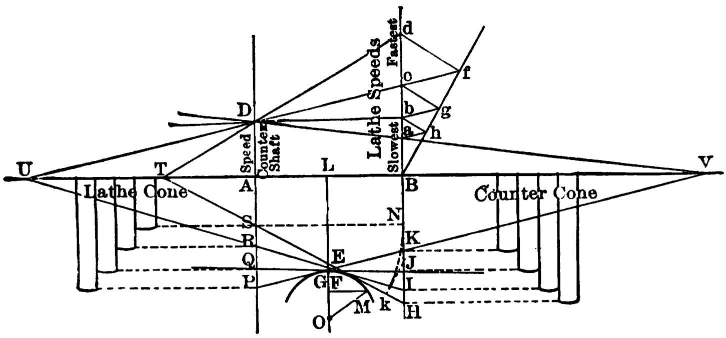


FIG 92.

spindle; draw Dd cutting AB at T; take AS as the diameter of the smallest cone-step, and draw TSH, thus obtaining BH as the diameter of the largest counter-cone-step.

Draw LO normal to AB midway between A and B, and with a radius $OM = .318AB$, draw the arc GM tangent to TH.

Bc may now be taken as the speed of the lathe-spindle for another pair of steps. To determine the diameters of these steps, draw cD cutting AB at U, and draw URI tangent to the arc GH. AR and BI are the required diameters of the steps.

If the steps just found are to give one speed of a geometrical series, of which Bd is the step for the fastest, the speed Bc can be found by the equation given above. The remaining speeds and steps can then be found by taking Bf at any convenient angle with Bd, drawing bf and cf, and then the lines cg, gb, bh, and ha parallel to df and cf respectively, as shown. The distances ab, bc, and cd will then be in geometrical ratio.

If any line, as Db, does not intersect AB on the drawing-board, the steps can be determined by the proportion

$$Bb : BJ :: AD : AQ.$$

83. Diagram for determining diameters of cone-pulley steps.— The ratio of the diameters of the steps of a pair of cone pulleys varies from unity, when the diameters are equal, to infinity, theoretically, when the radius of the larger equals the distance between their axes and that of the smaller is zero. While an infinite ratio can never exist practically, it is convenient to assume it in order to determine the curves of a diagram that can be used graphically for finding the diameters of pairs of steps which will require a constant length of open belt.

In order to make a single series of calculations give the radii for a variation of diameter ratios from unity to infinity, the radius of the largest step must be taken equal to the distance between pulley centres. This gives diameters so large for the intermediate steps that the sum of the radii of any two mates exceeds the distance between shaft centres, and, consequently, the complete circumference of such a pair of steps can not be placed on shafts so that the belt will run upon them. This apparently destroys the usefulness of the diagram. It is only apparently, however, for it will be shown later that it can be used to determine the diameters of any series of pairs of steps that can be placed in proper positions on the shafts, and will require the same length of open belt for every pair.

Any distance between centres of pulleys can be used for the calculations and diagram. For convenience it will be taken as 10 units.

The following notation is used:

d = distance between pulley centres;
R = radius of the larger of a pair of steps;
r = radius of the smaller of a pair of steps;
ρ = radius of each step when they are equal;
L = length of belt.

Put $R = d$; then

$$r = 0 \quad \text{and} \quad L = 2\pi R = 2\pi d = 2\pi 10 = 62.832.$$

Again, take $R = r$; then

$$L = \pi(R + r) + 2d = 2\pi R + 2d,$$

whence

$$R = \frac{L - 2d}{2\pi} = \frac{62.832 - 20}{2\pi} = 6.817 = r = \rho.$$

For the construction of a diagram for determining the diameters of pulley steps, draw a horizontal line OP, Fig. 93, and from P measure a vertical distance PQ of 10 units.

This is the radius of the larger step when the ratio of diameters is infinity. From O measure 6.817 units, this being the radius of

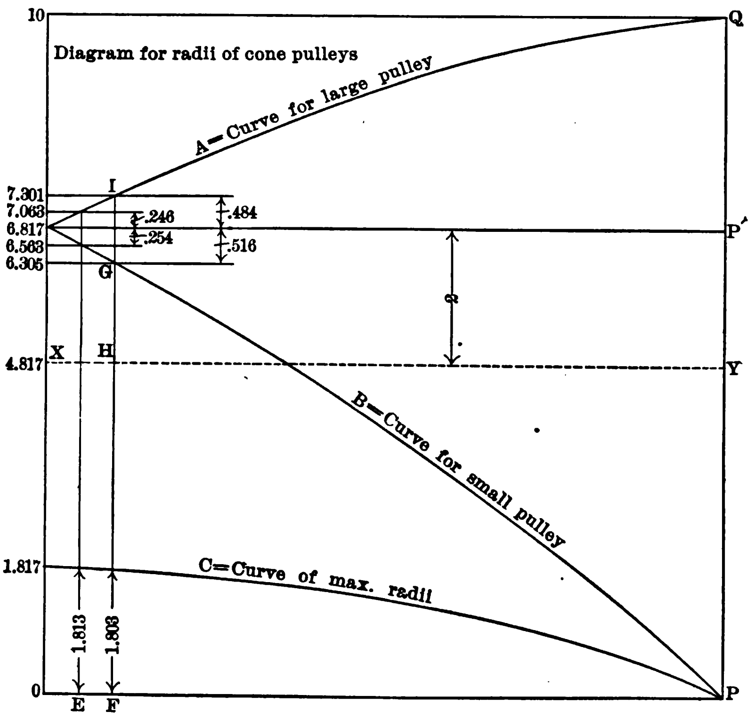


FIG. 93.

both pulleys when $R = r$ as found above. Intermediate points on the curves A and B can be found by assuming values for $(R - r)$ and solving for values of R and r that will keep L a constant, as follows:

Take $R - r = 0.5$; then, substituting in the equation for length of open belt, which is

$$L = 2\sqrt{d^2 - (R-r)^2} + \pi(R+r) + 2(R-r)\left(\text{arc } \sin^{-1}\frac{R-r}{d}\right),$$

which may be written

$$\pi(R+r) = L - 2\sqrt{d^2 - (R-r)^2} - 2(R-r)\left(\text{arc } \sin^{-1}\frac{R-r}{d}\right),$$

gives

$$\begin{aligned}\pi(R+r) &= L - 2\sqrt{100 - (0.5)^2} - 2(0.5)\left(\text{arc } \sin^{-1}\frac{0.5}{10}\right),\\ &= L - 2\sqrt{99.75} - \text{arc of } 2^\circ\ 52'\\ &= L - 2 \times 9.9875 - \frac{172}{21600}2\pi \qquad (2^\circ\ 52' = 172')\\ &= 62.832 - 19.975 - 0.5 = 42.807.\end{aligned}$$

Whence

$$R + r = 42.807 \div \pi = 13.626,$$

and since $R - r = 0.5$, $2R = 14.126$ by addition;

therefore $R = 7.063$; $R - \rho = 7.063 - 6.817 = 0.246$,

and $\rho - r = 6.817 - 6.563 = 0.254$.

At E on OP (a convenient value of OE being .05 of OP), erect a perpendicular and measure from E the distances that have just been found as values of R and r (i.e., 7.063 and 6.563). This gives a point on each of the curves A and B.

Other points are found in the same manner; thus, when $R - r = 1$,

$$\begin{aligned}\pi(R+r) &= L - 2\sqrt{100 - 1} - 2(1)(\text{arc } \sin^{-1}\tfrac{1}{10})\\ &= L - 2 \times 9.949 - 2 \text{ arc of } 5^\circ\ 44'\\ &= 62.832 - 19.898 - 0.2002 = 42.734.\end{aligned}$$

Whence

$$R + r = 42.734 \div \pi = 13.603;$$

and since $R - r = 1$, therefore

$$R = 7.301;\quad R - \rho = 7.301 - 6.817 = 0.484;$$

$$r = R - 1 = 6.301;\quad \text{and } \rho - r = 6.817 - 6.301 = 0.516.$$

These values are laid off from F as the others were from E, taking $EF = OE$ for convenience, and two more points of curves A and B are thus determined. Finally, when a sufficient number of points have been located, the smooth curves A and B are drawn through them. From the method of obtaining the curves it can be seen that any perpendicular to OP will intersect A and B at points whose distances from OP are the radii of a pair of steps whose belt length is the same as that of any other pair determined in the same way.

Thus far OP has been taken as the centre line of the cone pulleys. It will now be shown that any other line XY, parallel to OP and at a distance not greater than 6.81" above it, can be used as a datum line from which to measure the radii of pairs of steps whose centres are on XY, and whose belt length is constant.

To do this it is necessary to show that the radii measured on any normal to OP, as FH, will require the same length of belt as a pair of steps having the radius $\rho = YP'$.

The radii of a pair of steps are both measured on the same normal to XY, and are the distances from XY to the curves A and B.

On FI the difference of radii is $GI = R - r$ in the formula. This is a constant quantity for the line FI wherever the datum line XY may be taken.

The formulæ for open belt length are

$$L = 2(\pi\rho + d), \text{ and}$$

$$L = \pi(R + r) + 2\sqrt{d^2 - (R - r)^2} + 2(R - r)\left[\text{arc sin}^{-1}\frac{(R - r)}{d}\right].$$

In the last formula $R - r$ is a constant for the line FI. Therefore

$$2(\pi\rho + d) = \pi(R + r) + \text{constant},$$

whence

$$R + r = \frac{2(\pi\rho + d)}{\pi} - \text{constant}. \quad . \quad . \quad . \quad . \quad (3)$$

Adding the constant $(R - r)$ to both sides of the equation gives

$$2R = 2\rho + \left[\frac{2d}{\pi} + (R - r) - \text{constant,}\right]$$

in which the bracketed quantity is a constant; therefore

$$R = \rho + \text{constant.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

By subtracting $R - r$ from both terms of (3), it is shown in the same manner that

$$r = \rho - \text{constant.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equations (4) and (5) are true for all values of ρ, for the line XY was taken at *any* distance from the meeting point of the curves A and B. And since FI is *any* normal to OP, they are also true for all normals to OP. The diagram, therefore, can be used for any datum line between O and the intersection of the curves A and B.

The above demonstration may be expressed as follows: *For any given value of $R - r$, whatever the values of R and r, R is always a certain fixed amount larger and r another fixed amount smaller than the corresponding ρ, the value of d being constant.*

A numerical example may serve to make this more clear. Take XY at a distance 4.187 above OP. This makes $\rho' = 2 =$ distance from XY to the intersection of A and B, this distance being chosen simply as a convenient value.

Then $\quad L' = 2(\pi\rho' + d) = 2(2\pi + 10) = 32.5664.$

Take $\quad R' - r' = 0.5;$

then $$\pi(R' + r') = L' - 2\sqrt{100 - .25} - 2(0.5)\left(\text{arc sin}^{-1}\frac{0.5}{10}\right)$$
$$= 32.5664 - 19.975 - .0503$$
$$= 12.5411.$$

Therefore $\quad R' + r' = 3.992$; and, since $R' - r' = 0.5$,

$R' = 2.246;\ R' - \rho' = 2.246 - 2 = 0.246;$

$r' = 1.746;\ \rho' - r' = 2 - 1.746 = 0.254.$

The values 0.246 and 0.254 show the same increase and decrease of diameters from ρ' as when $\rho = 6.817$ measured from OP as a datum line.

For the practical application of the diagram, it is convenient to construct it by taking the horizontal line passing through the intersection of A and B as the zero line, and measure distances above and below it, as in Fig. 94.*

Since the sum of the radii of any pair of complete steps cannot exceed the distance between centres, another curve, C, showing the maximum radii of pulleys, can be drawn.

Thus, when the diameters are equal, $\rho = d \div 2 = 5$ for the largest complete pulleys, and one point of curve C is at a distance $6.817 - 5 = 1.817$ above O in Fig. 93. When $R - r = 0.5$, $R = 7.063$ and $r = 6.563$. The values of R and r must therefore be decreased by an amount equal to

$$\frac{7.063 + 6.563 - 10}{2} = 1.813$$

in order to give pulleys that will just touch each other. The distance 1.813 above E gives another point of the curve C; and so on for other points.

For any distance between centres having a ratio to $10 = x$ (i.e., $d \div 10 = x$), the reading of pulley radii as given on the diagram must be multiplied by x in order to obtain the actual size; and conversely, if the radius of the step is given for a pair of steps at a distance apart $= d = 10x$, the given radius must be divided by x to obtain the reading on the diagram. In general, the scale of the cone radii equals

$$\frac{QP \text{ measured on the diagram}}{\text{Actual distance between pulley centres.}}$$

84. Application of cone-pulley diagrams.—Fig. 94. Suppose we have given all the radii of cone α, one radius $M'N'$ of cone β, and the distance between their axes, to find the other radii.

First find the difference of the radii of $M'N'$ and its mate MN, which is

$$M'N' - MN = mm',$$

and then find the ordinate whose intercept between the curves A and B equals mm' to scale; then from the intersection of this or-

* It is believed that this method of construction is due to Professor Klein of Lehigh University.

dinate with the curve *B* lay off the distance *mn* = radius of *MN*; through *n* draw a horizontal datum line which represents the axes of the cones.

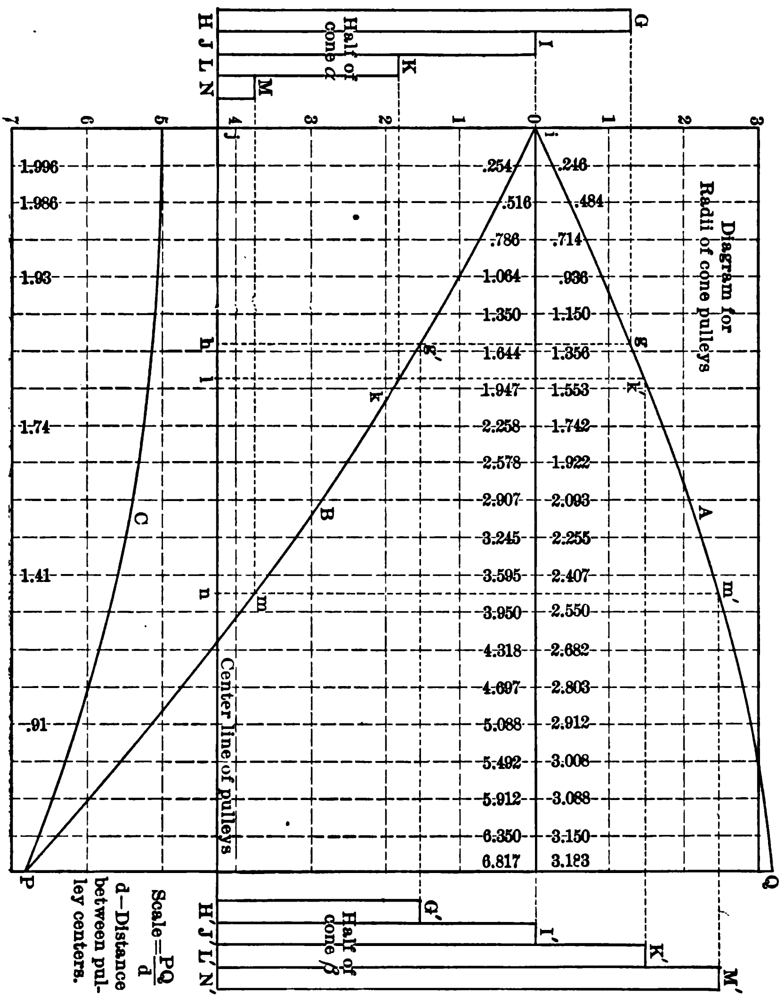


FIG. 94.

To find the mate for *KL*, determine the intersection *k* of curve *B* with a horizontal line at a distance *KL* above the datum line,

and through k draw the ordinate lkk'; then lk' is the radius required.

As shown in the figure, IJ and $I'J'$ are of the same radius. GH being larger than $G'H'$, the point g falls on A, and by drawing $gg'h$ the point g' is found on B.

Again, suppose the radii of one pair of steps are given, and the velocity ratios of the other pairs. Let the given velocity ratio of one pair be $a \div b$.

The datum line is located as before. Upon a piece of tracing-cloth or thin transparent celluloid draw a line TS, Fig. 95, and

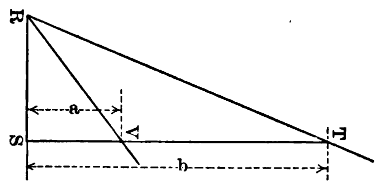


FIG. 95.

measure from S the distances SV and ST proportional to a and b; draw SR perpendicular to TS, and from any point R on SR draw RT and RV indefinitely prolonged. Now place the tracing-cloth on the diagram so that SR coincides with the datum line and move it along the datum line until the intersection of TR with A and VR with B lie on the same ordinate. The distances from these intercepts to the datum line are the required radii.

CHAPTER V.

CAMS.

85. It is frequently necessary to give some member of a machine a motion which cannot be gotten by any convenient system of linkages or other combinations having surface contact, or even with the aid of toothed gears having line contact.

In such cases the member may be driven with another which is shaped so as to produce the required motion of the first. The member thus shaped in accordance with the law of motion of the given part is called a **cam.** The driven member is the **follower.** On account of the nature of the engaging parts, they must have line contact.

In the majority of cases, the cam rotates or oscillates about an axis. The follower may have rectilinear motion, oscillate about an axis, or move according to some other law.

When the rotating member has a simple geometrical form, as a pin in a disk, corresponding to the crank-pin of an engine, and engages with an irregular part, the latter is commonly called an **inverse cam.** Such a device could be obtained in Fig. 12 by making the slot in which B travels of such a form as to give some particular motion to C. In general, this would require a slot of such a form that no sliding-block could be used on B. The contact between B and C would then be in a line.

Cams with Knife-edge, Pin, or Roller Follower.

86. General case.—This includes the follower having right-line motion, as in Fig. 96, and the rocker-arm follower, Fig. 97. The general case is represented in Fig. 98, where AE is the path of the knife-edge or roller axis. This path can be of any form within the limits of practical working. In the solution, the pitch curve of

the cam is found by assuming that the roller has no material diameter, or that a knife-edge follower is used; then the working curve

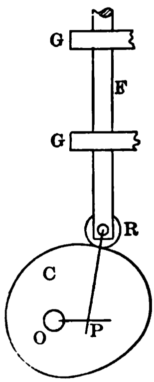


FIG 96.

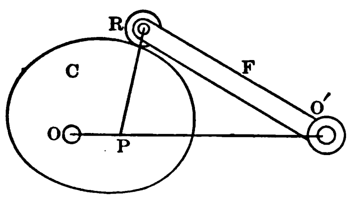


FIG. 97.

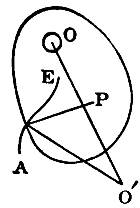


FIG. 98.

is determined from the pitch curve for the kind of follower to be used.

1st method.—In Fig. 99, *AE* is the path to be travelled by the knife-edge or axis of the roller of the follower, and *O* is the centre of

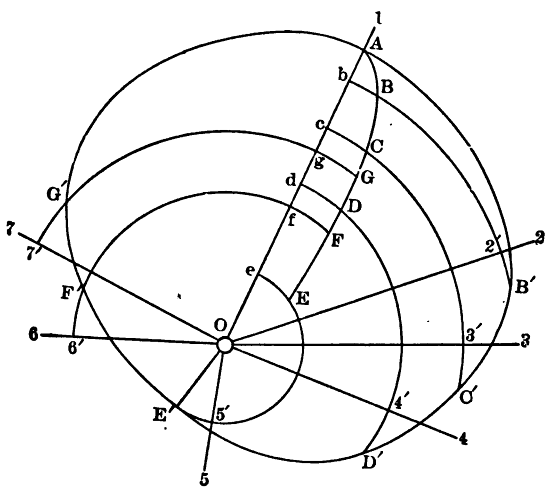


FIG. 99.

the cam. Through *A* draw a radial line *O*1, which can be used as a datum line for measuring the given angles 1*O*2, 1*O*3, etc., through

which the cam is to rotate for the corresponding given positions A, B, C, etc., of the follower. Assume that the follower is at A when the datum line is in the position shown. Then A is one point on the cam curve. With O as a centre and OB as a radius, strike the arc $B2'$ intersecting $O1$ at b and $O2$ at $2'$. From $2'$ measure off the arc $2'B' = Bb$. Then B' is another point on the pitch curve of the cam; for when the cam has rotated counter-clockwise, so as to bring $O2$ into the position $O1$, B' will lie at B. In the same manner with OC as a radius, draw the arc $C3'$ cutting $O3$ at $3'$ and take the distance $3'C' = Cc$. Then C' is another point in the curve of the cam. By continuing this operation for all the given positions of the follower and corresponding angles of rotation for the cam, a number of points are obtained through which a smooth curve can be drawn which is a pitch curve of the cam.

Since this is the curve against which a knife-edge or pin without material diameter would have contact, it clearly would not answer for a blunt point or roller such as is commonly used against the face of the cam. Suppose that a roller is to be used whose axis will be guided along the curve AE. Then the working curve of the cam must lie parallel to the pitch curve and at a distance from it equal to the radius of the follower roller. Such a curve can be obtained by describing a number of circles of a radius equal to that of the roller with their centres on the pitch curve, and drawing a smooth curve tangent to them.

If the follower is to be held against the cam by gravity, a spring or other device, only one working face is required on the cam, as in Fig. 96; but if the cam is to give a positive motion to a single roller, the latter can be guided in a groove formed in the face of a disk, the width of the groove being equal to the diameter of the roller, and its sides parallel to the pitch curve of the cam.

When the path of the follower axis is a straight line intersecting the axis of the cam, a line $O1$, drawn through A, will coincide with this line, and the distances Bb, Cc, etc., become zero, and the points on the pitch curve of the cam obtained by following the same solution as given above will lie on the lines $O2$, $O3$, etc.

2d method.—A solution embodying exactly the same principles as the preceding one can be made by drawing upon a piece of tracing-cloth or transparent sheet celluloid the radial lines $O1$, $O2$, $O3$,

etc., at the given angles, then pinning the meeting point of these lines at O, so that $O1$ on the tracing-cloth will coincide with $O1$ on the drawing-paper, and marking a point on the tracing-cloth immediately over A on the paper. This is one point on the cam curve. The tracing-cloth can then be rotated about O so that $O2$ on it will coincide with $O1$ on the drawing-paper, and a point marked on the tracing-cloth immediately over B. This is another point on the cam curve. Rotating again and marking gives another point, and so on throughout their entire rotation. The smooth curve drawn through the points on the tracing-cloth is the pitch curve of the cam.

The points on the tracing-cloth can, of course, be transferred to the drawing-paper by placing the former so that the line $O1$ of one will coincide with $O1$ of the other, and perforating the tracing-cloth through the points determined on it, thus transferring them to the paper. This method is generally more rapid than the first.

87. Velocity ratio of cam and follower.—Since the virtual centre of the cam and follower must have the same velocity when considered as a point in either, it is only necessary to locate this point in order to obtain a basis for obtaining the linear velocity of any other point, or the angular velocity ratio of the cam and follower.

Thus, in Fig. 96, the centre of the cam and follower lies at P, which is on the intersection of a line through the axis of the cam at right angles to the path of the follower with another line through the axis of the follower normal to the cam curve. Then, since all points in the follower, the rotating roller excepted, have the same linear velocity, any point at the radial distance OP from the axis of the cam has the same linear velocity as all parts of the follower.

In Fig. 97, P lies at the intersection of the line of centres of the cam and follower with a normal to the cam curve at the axis of the roller. The angular velocities of the cam and follower have the ratio

$$V^{\circ}C : V^{\circ}F = O'P : OP.$$

In Fig. 98 it is necessary to know the radius of curvature of the pitch curve of the cam where the roller is at any instant, in order to locate the centre of the cam and follower. When the radius of

curvature is known, the centro O', about which the axis of the roller is moving at that instant, lies on a normal to the pitch curve at the axis of the roller, and at a distance equal to the radius of

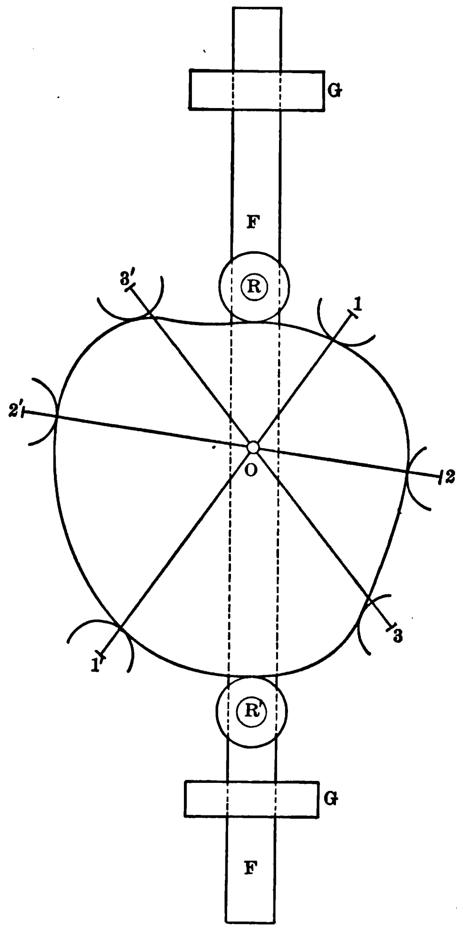


FIG. 100

curvature from the pitch curve. The point P is then located as in Fig. 97, and the velocity ratio is found in the same way.

88. Cam with reciprocating follower having two lines of contact on opposite sides of the cam.—When the motion of the

follower for one half a revolution of the cam is to be exactly the reverse of that for the other half-revolution, positive motion can be obtained by using two rollers R and R', Fig. 100, of the same diameter, both attached to the follower which extends across the axis of the cam. The curve for one half a revolution can be obtained for one roller by either of the methods of § 86. The starting point may be taken at any position of the follower.

Assume that the part of the cam R, 1, 2, 3, R', has been determined. To find the other part, suppose that R has moved to 1; then R' must lie at 1′ on the opposite side of O, the distance 11 being equal to RR', both measured between the axes of the rollers. The same method applies to any number of positions of R. The curve drawn tangent to the positions of R' at 1′, 2′, 3′, etc., completes the profile of the cam.

Cams Engaging with Plane Surface Followers.

89. General case.—This includes the special cases Fig. 101, where the follower F has a rectilinear motion, and Fig. 102, where it is a rocker arm.

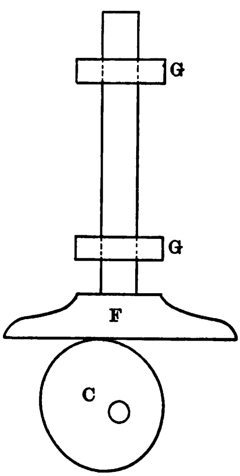


Fig. 101.

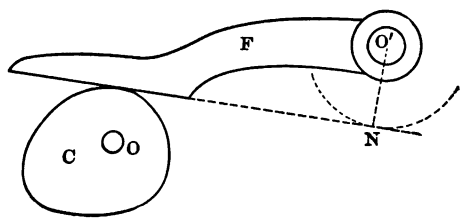


Fig. 102.

Fig. 103 is the general case. 1, 2, 3, etc., are the given positions which the follower is to occupy simultaneously with cor-

responding given positions of the cam. Draw $O1'$ perpendicular to 1, and intersecting it at A; then A is one point on the cam curve if $1A$ is the position nearest to the axis of the cam, that the follower is to occupy; for, if contact between them were at any other point, rotation of the cam in the proper direction would allow the follower to come nearer the axis of the cam. B, C, D,

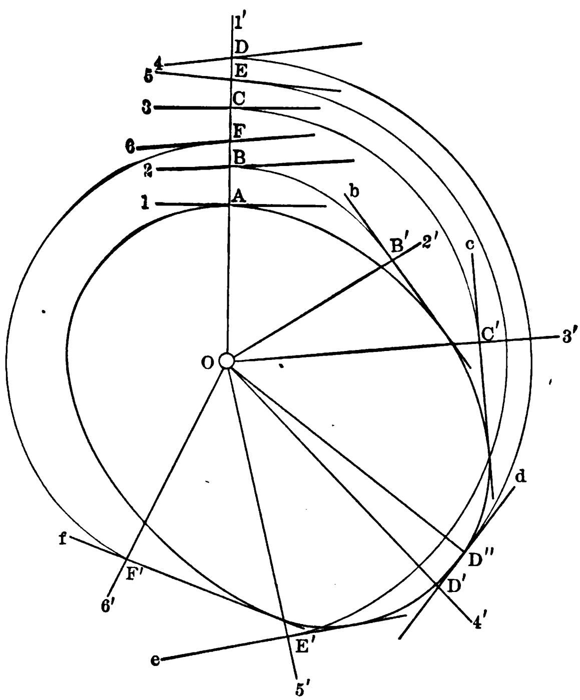


Fig. 103.

etc., are the intersections of $O1'$ with the other given positions of the follower.

Draw $O2'$, $O3'$, etc., at the angles $1'O2'$, $1'O3'$, etc., through which the cam is to rotate for bringing the follower into the positions 2, 3, etc. With O as a centre and radius OB, draw an arc cutting $O2'$ at B', and through B' draw $B'b$ making the angle $OB'b = OB2$. Repeat this operation by drawing the arc CC' and

the line $C'c$ at the angle $OC'c = OC3$; and so on for all the given positions of the parts.

If $4D$ is one limit of the motion of the follower, then OD'', drawn at right angles to $D'd$, locates D'' as another point on the curve. The reason that D'' is a point on the curve is similar to that for A.

A smooth curve passing through A and D'', and tangent to all the lines $1A$, $B'b$, $C'c$, etc., is the cam curve. If any of the lines $1A$, $B'b$, $C'c$, etc., is cut out by the others so that it is impossible to draw a curve tangent to it without intersecting some of the other lines, then the problem cannot be solved without moving O farther away from the follower. This makes a larger cam, of course.

The solution by the use of tracing-cloth or sheet celluloid can be made as in the second method of § 86. The positions 1, 2, 3, etc., of the follower, and of $O1'$, the normal to 1, are drawn on the paper as before. $O1'$, $O2'$, $O3'$, etc., are then drawn on the tracing-cloth, which is pinned to the draughting-board at O and turned so that $O1'$ on it lies over $O1'$ on the paper. A line is then drawn on the tracing-cloth to coincide with $1A$ on the paper. The tracing-cloth is then turned to bring $O2'$ over $O1'$, and $B2$ is traced on the cloth. The same process is repeated for all given positions of the follower, and the cam curve drawn on the cloth tangent to $A1$, $B2$, $C3$, etc.

For the mechanism of Fig. 101, the lines 1, 2, 3, etc., of Fig. 103 are all parallel, and the angles $OA1$, $OB'b$, $OC'c$, etc., are all right angles. The point D'' therefore falls on D'. This is true both when the face of the follower is normal to the direction of motion, as shown by Fig. 101, and when inclined at any angle to the direction of motion.

When the plane of the face of the rocker-arm follower, Fig. 102, is at a distance $O'N$ from the axis of oscillation, the profile line of the face must be tangent to a circle of radius $O'N$ with O' as a centre, for all positions of the follower. The radius of this circle becomes zero when the plane of the face passes through the axis of the follower, and all profile lines pass through O'.

In designing flat-faced followers, care should be taken to see that the face is long enough for the full sweep of the line of contact over it.

90. Cam engaging with two parallel plane surfaces.—When the motion of the follower is to be rectilinear, as in Fig. 104, the curve for one half a revolution of the cam can be assumed, or determined according to some given motion of the follower, and the other half obtained to conform with it. In Fig. 104, suppose that the portion of the curve TST' is given, T and T' being the points of tangency for the positions of the full parts as indicated by the full lines. Then the remaining portion of the cam curve

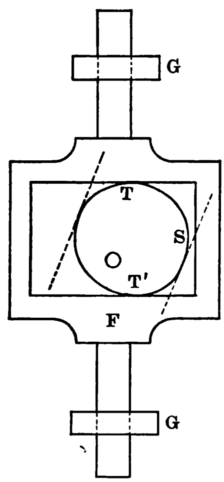


FIG. 104.

can be obtained by placing the follower in various positions tangent to the given part of the curve, one being shown by the dotted lines, thus obtaining a series of intersecting straight lines to which the remaining portion of the curve can be drawn tangent. T and T' must lie on the same normal to the faces of the follower.

When a complete cam is to be designed for certain positions of the follower and the corresponding angular rotations of the cam, the solution is practically the same as above, the only difference being that in this case the follower must be placed with its faces at the given distances from the axis of the cam for the given angular positions of the cam. An envelope is thus formed to which the cam curve must be drawn tangent.

For a rocker-arm follower, as shown in Fig. 105, the operation is a similar one. It should be noted, however, that in this case the half-revolution of the cam must be taken relatively to the follower and not to the member of the machine joining them.

91. General solution for cam with any form of follower having one line of contact.—While this includes all the preceding cases, it is hardly necessary to consider it in connection with them, but

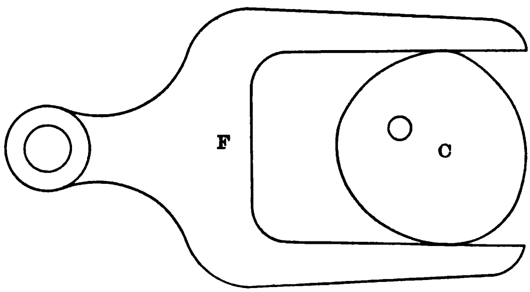


FIG. 105.

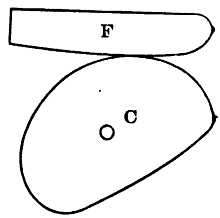


FIG 106.

rather as applicable to such problems as have not already been considered.

In Fig. 106 the follower F may have any form of face as shown, and any kind of motion.

In Fig. 107 the given positions of the profile of the face of the follower are represented at 1, 2, 3, etc., and the centre of the cam at O. Draw any line $O1'$, intersecting 1, 2, 3, etc., at A, B, C, etc., and take $O1'$ as the datum line from which to measure the angles $1'O2'$, $1'O3'$, etc., through which the cam is to pass to bring the follower into the positions 2, 3, etc. With O as a centre, and radius OB, strike the arc BB' cutting $O2'$ at B', and through B' draw the curve Bb in the same relative position to $O2'$ as $B2$ is to $O1'$. In the same way determine the position $C'c$ corresponding to $C3$. Continuing this operation for all the given positions of the follower and corresponding angles of rotation of the cam, an envelope is formed, and the cam curve can be drawn tangent to all the lines $B'b$, $C'c$, etc.*

* For a full treatment of cams engaging with two or more plane surfaces, see Reuleaux's "Kinematics" and Robinson's "Principles of Mechanisms."

92. Involute cam for stamp-mill.—Fig. 108. AB is the centre line of the stamp-rod, and D the centre of the cam-shaft and cam, the two shafts being at right angles. The stamp is raised by the engagement of the cam surface EF with the lower surface of a drum or disk C attached to the stamp-rod. As soon as the cam rotates enough for the point F to pass out of contact with C, the stamp falls by gravity, and is again lifted by the cam. Two or more arms similar to EF are commonly used to engage with C,

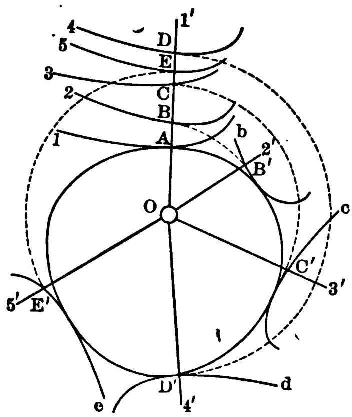


FIG. 107.

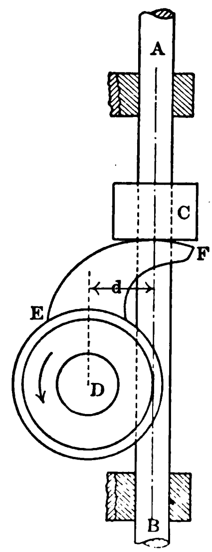


FIG. 108.

thus reducing the speed of rotation of the cam for a given rate of working the stamp.

The curve EF is frequently made of the involute form, the base circle having a radius equal to the distance between shaft centres, in order to make the path of contact a right line perpendicular to the lower face of C. Thus, in the figure, d is the distance between shaft centres and also the radius of the base circle of the involute. The path of contact lies in a plane which passes through AB and is perpendicular to the paper.

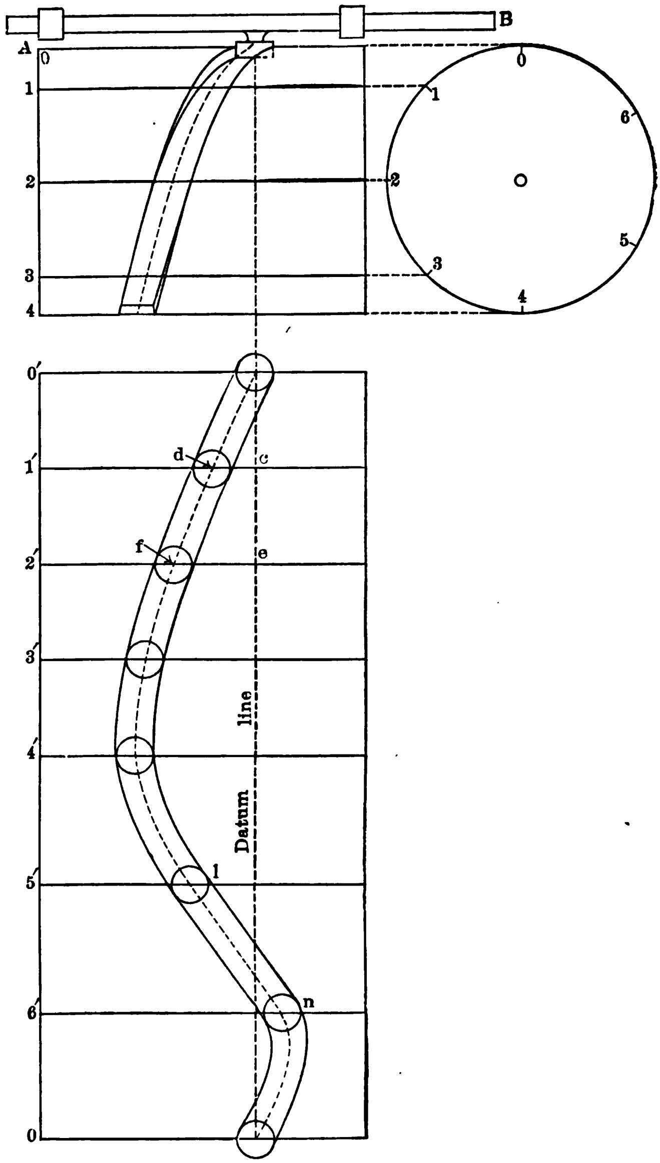


FIG. 109.

CYLINDRICAL CAMS.

93. Cylindrical cam with follower having a right-line motion.—Fig. 109 shows the end and side views of a cylindrical cam composed of a drum with a groove cut around it, which reciprocates the follower AB by means of the roller or pin running in the groove. The lower part of the figure is the development of the surface of the drum. The problem is to find the curve of the groove that will give the follower certain definite positions at different parts of a revolution of the cam.

In the development of the cylindrical surface, take the projection of a circle of the drum passing through the initial position of the centre of the pin as a datum line, and from it lay off on $1'$ the distance cd, which is the given travel of the follower while the cam is turning through the arc $O1$; then d is a point of the developed curve. In the same manner lay off on $2'$ the distance ef, which is the travel of the follower while the cam rotates through the arc $O2$, thus obtaining another point of the curve. When a sufficient number of points have been obtained, a smooth curve drawn through them will give the centre line of the cam-curve development. If, during any part of the cam's rotation, as from 5 to 6, the follower is to have a uniform motion, the points 1 and n on $5'$ and $6'$ are joined by a straight line. The curves of the edges of the grooves are obtained by drawing circles of the diameter of the follower-pin with their centres on the centre line of the curve and then drawing smooth curves tangent to the circles.

94. Cylindrical cam and rocker-arm.—Fig. 110. In this case a pin or roller on the rocker-arm ce engages with a groove in the surface of the cam as before. Take ce as the initial position of the rocker-arm, and ca'' the position after the cam has turned through the arc oa.

The element a is shown in the development of the cam at a'. The centre of the follower-pin does not fall on a', however, but at a distance $d' = d = d''$ ahead of it, d'' being the versed sine of the angle through which the follower has turned. The point g on the cam curve lies at the intersection of the circle passing through a'', with the cam element lying at a distance d' in advance of a'. Other points on the cam curve are obtained in the same manner.

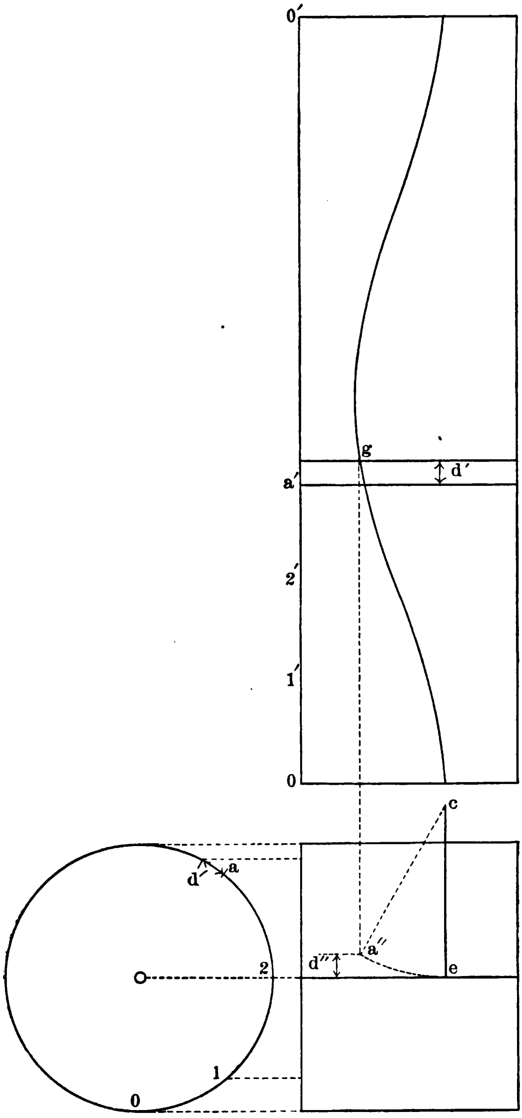


FIG. 110.

Since the rocker-arm swings so that the axes of the cam and follower-pin extended do not intersect except for one position of the follower, the pin will not fit the groove exactly. The error is slight and negligible for ordinary practice when the angle of the rocker-arm is not great. This angle can increase with the ratio of the diameter of the cam to the length of the rocker-arm. Strictly speaking, the cam surface is not a circular cylinder when a rocker-arm follower is used, but within the ordinary limits of practice this does not need consideration.

CONICAL CAMS.

95. In Fig. 111, *Q* is the side elevation and *T* the end view of a truncated cone upon which a cam curve that will give the follower,

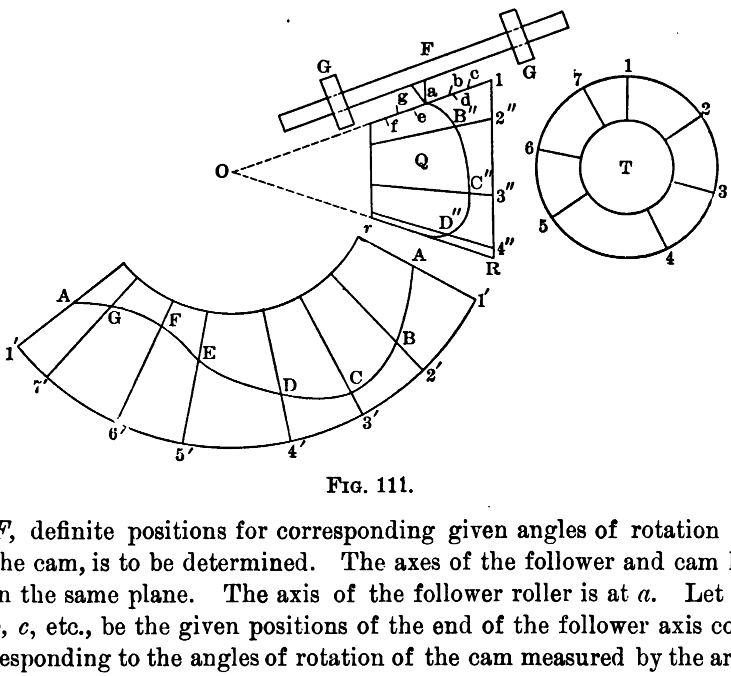


FIG. 111.

F, definite positions for corresponding given angles of rotation of the cam, is to be determined. The axes of the follower and cam lie in the same plane. The axis of the follower roller is at *a*. Let *a*, *b*, *c*, etc., be the given positions of the end of the follower axis corresponding to the angles of rotation of the cam measured by the arcs 12, 13, 14, etc.

With the apex *O* of the conical surface as a centre, and radius *OR*, strike the arc 1′1′, making the length of the arc equal to the

circumference of the circle of the larger end of the cam; and with the same centre and Or as a radius, strike another arc lying between the radial lines $O1'$ and $O1'$, as shown. The area thus inclosed represents the developed surface of the conical cam blank. The distances $1'2'$, $1'3'$, etc., equal 12, 13, etc., measured on the arcs of the circles. Take $OA = Oa$. Then A is one point in the curve on the developed surface, and lies upon the element 1 of the end view. Take $OB = Ob$, upon $12'$. Then B is another point in the path of the axis of the cam follower. Continuing in this way, all the points are determined for the given positions of the follower axis. If the developed surface is wrapped back upon the cam blank, the curve will take the form shown upon it.

For a pin or roller of sensible diameter, the walls of the groove are obtained as in § 93.

If a rocker-arm follower similar to that for the cylindrical cam of § 94 is used, allowance must be made in the same manner for the displacement of the axis of the following roller from the cone element representing the angle through which the cam has rotated from its initial position.

With the rocker-arm follower, the surface of the cam is not strictly conical, but ordinarily its deviation from the conical form can be neglected.

INVERSE CAMS.

96. Inverse cam with rectilinear motion.—Fig. 112 represents an inverse cam C which has a reciprocating motion through the guides GG. O is the driving-shaft, and R is the roller or pin which engages with the cam to drive it. In designing such cams, the position of the cam is generally given for corresponding positions of the driving-pin.

In Fig. 113, $S'T'$ is drawn parallel to the motion the cam is to have. O' is the axis of the driving-shaft and $O'R'$ is the length of the driving-arm. The axis of the pin travels in the circle of radius $O'R'$. When the pin is at R', on a line passing through O' and normal to $S'T'$, it is at one end of the slot; and when a half-revolution has been made, bringing the pin to N', it is at the opposite end. The cam curve can, therefore, be designed for given positions of the cam for corresponding positions of R' embracing only

180° of the circle. These positions of R' can be partly above and partly below $R'N'$, however. Thus, starting from R', positions of the cam may be given for 120° of the circle, and then no given law of motion required until R' has reached N', from which point positions of the cam may be given for the remaining 60° of the 180°. No two given positions of R' can be symmetrically located with respect to $R'N'$. When the given positions of R' lie on dif-

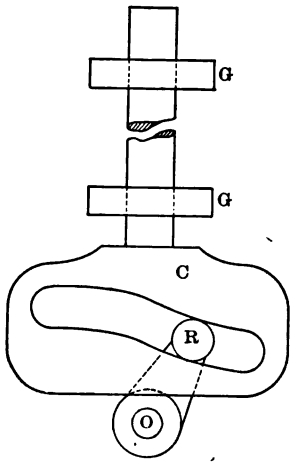


FIG. 112.

ferent arcs of the circle, the design can not be a general one, for the points on the cam curve must come in such a relative position that a smooth curve can be drawn through them.

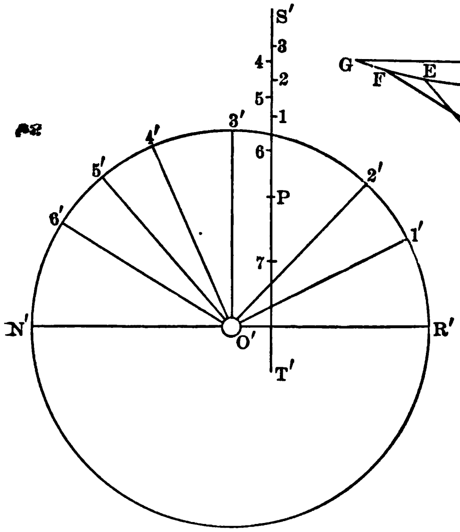
Let P, 1, 2, etc., be the given positions of any point P, in the cam, which travels along $S'T'$, and R' 1', 2', 3', . . . N' the corresponding positions of the pin. Two methods of designing the cam are applicable.

1st method.—Tracing-cloth can be applied in this case as in all others. Since it aids in the description of the solution made directly on the paper, it will be given first.

Draw a straight line on the tracing-cloth in any convenient position. Place the cloth so that the line upon it coincides with $S'T'$, and mark the points R' and P upon it. Now move the cloth so that the point last marked lies at 1, keeping the straight line upon the tracing-cloth coincident with $S'T'$, and mark 1' on

the cloth. Now move the tracing-cloth until the first point marked upon it lies over 2, and mark 2′ upon it. Doing this for all the given positions locates the required points on the curve.

2d method.—This assumes that the cam remains stationary and the axis of the driving-shaft moves. In Fig. 114, draw ST parallel



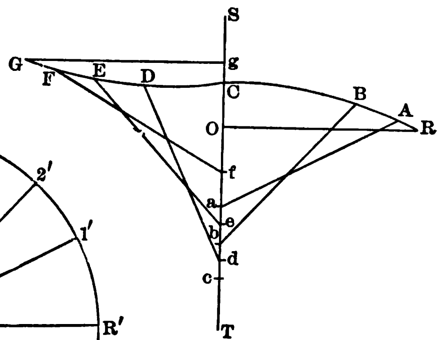


Fig. 114

Fig. 113.

to $S'T'$ of Fig. 113, and from any point O draw OR parallel and equal to $O'R'$; then R is one point on the cam curve. Take $Oa = P1$, and make aA parallel and equal to $O'1'$, thus obtaining another point A on the curve. For a third point, take $Ob = P2$ and draw bB parallel and equal to $O'2'$, which locates B on the required curve. And so on for all the remaining points.

97. Oscillating inverse cam.—Fig. 115 is a mechanism in which

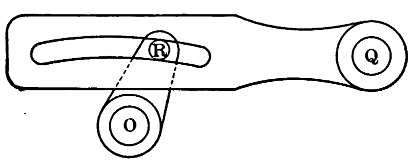


Fig. 115.

the inverse cam C oscillates about the centre Q. O is the axis of the driving-shaft and R the axis of the roller. The design of the

cam when simultaneous positions of it and the driving-arm are given, can be accomplished as follows: In Fig. 116, O' is the axis of the driving-shaft, and Q' that of the rocker-shaft of the inverse cam. By drawing $N'Q'$ through O', and a circle with centre O' and radius equal to that of the driving-arm, the points R' and N',

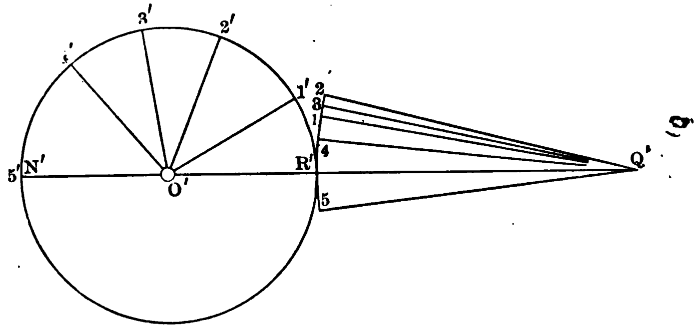


FIG. 116.

which indicate the positions of the driving-pin when it is at the ends of the slot, are determined. (See § 96 for the limits of the motion of the driving-arm for which the cam can be designed.) $Q'R'$ may be taken as the datum line for motion of the cam, and $O'R'$ the same for the driving-arm. $Q'1$, $Q'2$, $Q'3$, etc., and $O'1'$, $O'2'$, $O'3'$, . . . N' are given positions of the cam and driving-arm.

1st method.—By using tracing-cloth, the cam curve can be obtained by pinning the cloth on the draughting-board at Q', and marking a point R' upon it. It can then be swung about Q' so that the point just marked lies at 1, and the point $1'$ marked upon it. Continuing this for the remainder of the given positions locates points of the cam curve upon the tracing-cloth.

2d method.—To solve without the use of the transparent

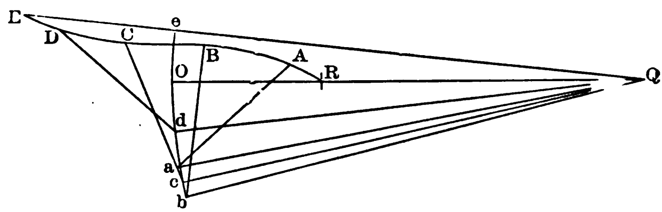


FIG. 117.

cloth, the method of Fig. 117 can be used, together with Fig. 116.

This assumes that the axis O', of the driving-shaft, moves, and the cam remains at rest. In Fig. 117, Q is the centre of the cam axis and be is an arc of radius $Q'O'$ as given in Fig. 116. Draw OQ, and take $OR = O'R'$; then R is a point on the required curve. For a second point draw Qa, making the angle $OQa = O'Q'1$; then draw aA so that the angle $QaA = Q'O'1'$, and the length $aA = O'1'$; then A is another point on the curve. Similar operations locate the remaining points B, C, etc., through which the curve can be drawn.

98. Inverse cam having any motion.—In Fig. 118, PS is the path to be traveled by a given point P in an inverse cam. In order to show the motion of this cam in the general case, a reference line PQ can be drawn upon it. Let the given positions of this reference line be PQ, $1a$, $2b$, etc. The centre of the driving-shaft is at O, Fig. 119. The axis of the roller R travels in the circle $R1'3'$. The

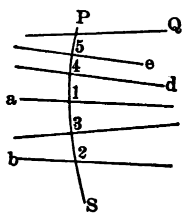


Fig. 118.

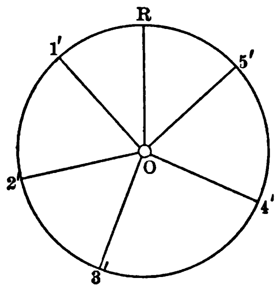


Fig. 119.

driving roller is to be at R when the reference line of the cam is at PQ. The angles of rotation of the driver corresponding to the given positions $1a$, $2b$, etc., of the cam, are $RO1'$, $RO2'$, $RO3'$, etc.

In order to obtain the points of the curve, draw upon a piece of tracing-cloth a straight line and place it over the paper so that the line will coincide with PQ, marking both the points R and P. Now move the tracing-cloth so that the point last marked lies over 1, and the reference line coincides with $1a$, and mark the point coinciding with $1'$. Repeating this operation for the other given positions of the parts locates a series of points upon the tracing-cloth through which the curve of the inverse cam can be drawn.

The solution without the aid of the tracing-cloth requires so much more work, and this form of cam is so unusual, that it is not thought necessary to give it here.

CHAPTER VI.

PARALLEL AND STRAIGHT-LINE MOTIONS.

99. Definitions.—Strictly speaking, a parallel-motion mechanism is one which has at least two points which move in parallel paths. There are numerous mechanisms for obtaining the motion of a point in a straight line, or approximately so, by the use of turning pairs, or sliding pairs not connected directly to the link in which the point lies. They are also called (somewhat, inappropriately) "parallel motions." Some of these mechanisms are frequently made use of in practice, those consisting of turning pairs only, being more serviceable and readily constructed.

100. The Parallelogram is used in connection with some of the straight-line motions. It gives a truly parallel motion.

Fig. 120 is a parallelogram having only one point, the articulation of a and b, fixed, in this case at O. Take any point C on the

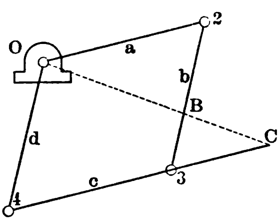


FIG. 120.

link c and draw OC, cutting b at B; then, from similar triangles,

$$\frac{B3}{O4} = \frac{C3}{C4}, \text{ or } B3 = \frac{C3}{C4} O4 = \text{a constant.}$$

Therefore, B is always at the same point on b for a given position of C on the link c.

Again,

$$\frac{OC}{OB}=\frac{4C}{43}$$

and is, therefore, constant for all positions of the mechanism.

It follows from the above that if C is made to travel over any path, an exactly similar path will be traced by B upon a smaller

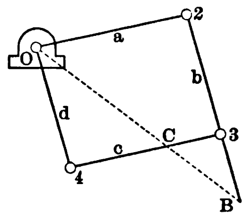


FIG. 121.

scale. By taking C as in Fig. 121, the path travelled by B will be greater than that of C. Any ratio of size can be obtained by moving C along the link c, the ratio always being

$$CO : BO.$$

The fixed point may be taken anywhere on any link, the only condition to be fulfilled being that the fixed point and the two tracing points must lie on a straight line.

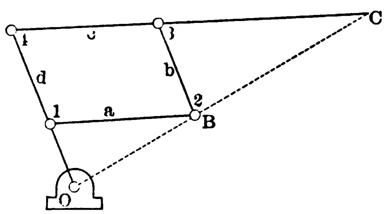


FIG. 122.

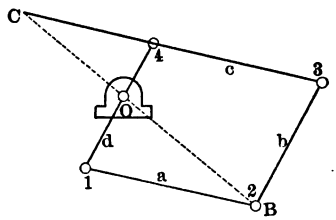


FIG. 123.

Figs. 122 and 123 show different locations of the fixed point. In any of the figures, C can be taken at any other point on c.

The parallelogram is frequently used as a "pantagraph" for enlarging, reducing, or copying drawings.

101. Watt's parallel motion, Fig. 124, is the most universally used of all approximately parallel motions. In the best and most

common form of the mechanism the describing point P is at the middle of the link a, whose length is about equal to the stroke of P.

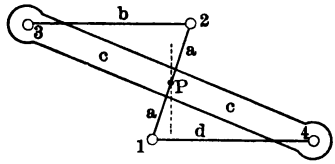


Fig. 124.

At the middle of the stroke, d and b are parallel to each other and perpendicular to the path of P. The points 1 and 2 deviate to the right and left of the path of P by equal amounts at the middle and end of the oscillations. When the above conditions can not be obtained, they should be as nearly approximated as possible.

The path of P is a portion of the lemniscoid, which resembles a distorted figure 8. The part that is used in practice is wavy, but a small enough portion of it is taken to keep the deviation from a straight line within very small limits.

The data in accordance with which the mechanism must usually be designed in practice, as in the case of a steam-engine with a walking-beam whose piston-rod is guided by the Watt motion, are the

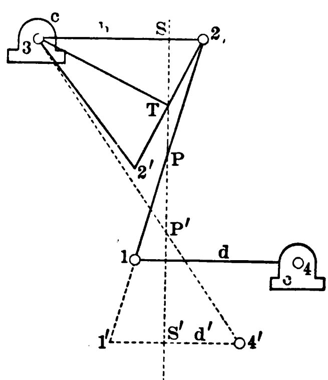


Fig. 125.

position of P at the middle of the stroke, and the location of the point 3 (or it may be 4). Having these data given in Fig. 125, the

length of b is found as follows: Draw $3S$ perpendicular to the path of P and intersecting it at S, take ST equal to one fourth of the stroke, join T and 3, and draw through T a perpendicular to $T3$, intersecting $3S$ at 2; then 32 is the required length of b, and $2P$ will be a part of the link a in its mid-position. By making $T2' = T2$, the position $32'$ of b at one end of the stroke is found. If P is at the middle of a, 1 is found by producing $2P$, making $P1 = 2P$; 4 is located by drawing 14 parallel to b and of the same length.

If 1 is at a given point $1'$, so that $P1'$ is not equal to $2P$, the best construction to make the point 1 fulfil the conditions prescribed for 2 is to take $S'P' = SP$, and draw $3P'$ to obtain $4'$ as shown in the figure. The link d then becomes d' with a length $1'4'$ and centre of oscillation at $4'$.

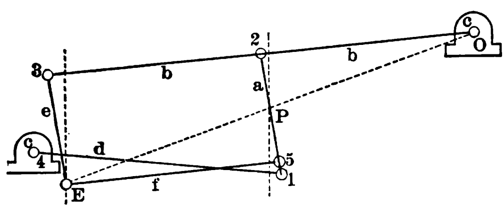


FIG. 126.

Fig. 126 is the combination of Watt's motion and the parallelogram, as generally applied to beam engines. The device occupies less space than the simple Watt motion applied for the same object. In this mechanism the Watt motion consists of a portion, $O2$, of the beam b, together with the links a and d. The middle point P of the link a, whose length is 21, moves in the vertical dotted line through P. The point E, which lies on OP and is an articulation of the parallelogram $23E5$, moves in a path parallel to that of P (see § 100). The piston-rod is attached at E, and a pump-rod at P, generally. In practice the point 2 is commonly at the centre of the beam and 1 coincides with 5, d being equal to one half the length of the beam.

102. The Peaucellier cell, composed of six links, Fig. 127, gives, with the aid of two additional links of equal length, a and b, Fig.

128, an accurately parallel motion to P, which follows the right line PP' when the link a is kept stationary and b oscillates about E. Four of the links of the cell form a rhombus and the remain-

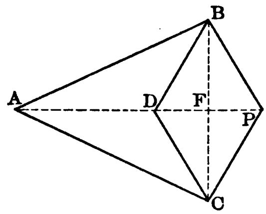


FIG. 127.

ing two are equal, so that in the complete mechanism there are four links of one length, two of another, and two more of a third length.

The proof that P travels in a straight line is as follows: In Fig. 127, F is the centre of the rhombus, and, on account of the equality and symmetry of the links, the points A, D, and P must lie in a straight line passing through F for any position of the mechanism; also, BC passes through F and is perpendicular to AP; therefore,

$$\overline{AB}^2 = \overline{AF}^2 + \overline{BF}^2$$

and

$$\overline{BP}^2 = \overline{PF}^2 + \overline{BF}^2,$$

from which

$$\begin{aligned}\overline{AB}^2 - \overline{BP}^2 &= \overline{AF}^2 - \overline{PF}^2 \\ &= (AF - PF)(AF + PF) \\ &= AD \times AP = \text{a constant,}\end{aligned}$$

for AB and BP are both constants.

Now, in Fig. 128, if P and D move to the positions P' and D', then, in accordance with the above equation,

$$AD \cdot AP = AD' \cdot AP',$$

whence

$$\frac{AD}{AD} = \frac{AP'}{AP}.$$

The triangles ADD' and $AP'P$ must, therefore, be similar. But the points A, D, and D' lie on the circumference of a circle

whose diameter is AD'; therefore the angle ADD' is a right angle and the angle APP' must also be a right angle. Since this is true

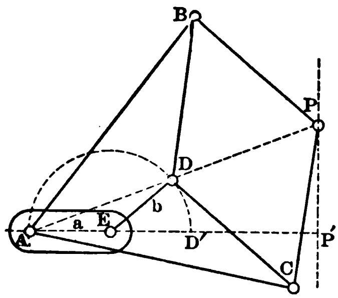


FIG. 128.

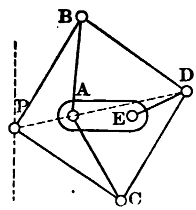


FIG. 129.

for any position of the mechanism, it must be true for all positions. Therefore P must move in a straight line PP' perpendicular to AE.

Fig. 129 is another position of the mechanism. The same proof applies to it.

103. Grasshopper parallel motion.—When the ordinary slider-crank chain has the connecting-rod b and crank a, Fig. 130, of equal

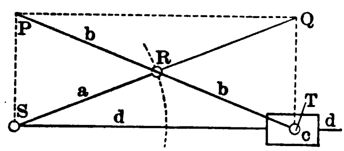


FIG. 130.

lengths between their articulations, a point P on the connecting-rod at a distance $PR = RS = RT$ from R will move in a straight line perpendicular to d; for the instantaneous centre of b is at Q, and $QT = PS$; therefore P moves perpendicularly to d, and PS is the path of P.

An approximation of this motion is obtained by making the slide c into a link, Fig. 131, whose centre of oscillation is so placed that a line passing through it parallel to the path of P bisects the angle formed by the extreme positions of c. Since T has but a slight

motion for a considerable stroke of P, the deviation of P from a straight line is but slight when c has a length large in proportion

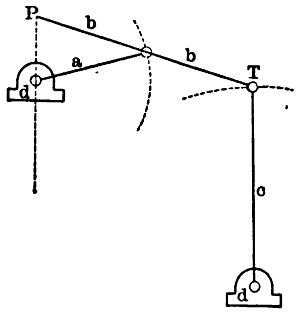


FIG. 131.

to the other links, thus making the path of T a very flat arc of a circle.

104. Steam-engine indicator motions.—Most of the steam-engine indicator motions are modifications of the Grasshopper straight-line motion described in the preceding paragraph. The mechanism of the Tabor Indicator is shown in Fig. 132. The pencil-point P is

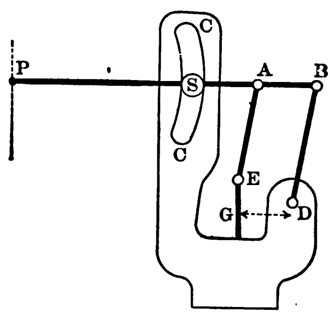


FIG. 132.

guided in a straight line by a stud S which moves in the cam curve CC. The form of this curve can be determined by guiding the point P along a straight edge, and tracing the curve with a point at S. Having obtained this curve, the cam can, of course, be made to give the pencil a straight-line motion. D is the stationary point

and E is the articulation of the piston-rod with the driving-link which connects to the pencil-arm at A.

The proportions of the Tabor Indicator as made by the Ashcroft Manufacturing Company are: *

$$PB = 3\tfrac{1}{8}''; \; SA = \tfrac{5}{8}''; \; AB = \tfrac{5}{8}''; \; GD = \tfrac{9}{16}''.$$
$$AE = 1''; \; BD = 1\tfrac{1}{4}''.$$

The range of the pencil-point is $3\tfrac{1}{8}''$.

The Thompson Indicator motion is shown in Fig. 133. D and F are the stationary points, and E is the articulation of the piston-

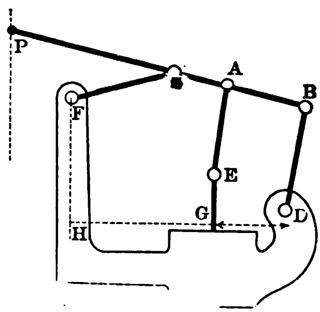


FIG. 133.

rod and link AE, connecting it to the pencil-arm at PB. The proportions of the Thompson Indicator as made by Schäffer & Budenberg are:

$$PB = 3.18''; \; SA = .56''; \; AB = .795''; \; BD = 1.17'';$$
$$FS = 1.03''; \; GD = .72''; \; GH = 1.47''.$$

As made by the American Steam Gauge Company the proportions are:

$$PB = 3''; \; SA = \tfrac{1}{2}''; \; AB = \tfrac{3}{4}''; \; BD = 1.5'';$$
$$FS = \tfrac{15}{16}''; \; GD = .78''; \; GH = 1.32''; \; DH = 2.1''.$$

The Crosby indicator is represented conventionally in Fig. 134. The pencil-point is at P in the rigid link PB, to which are attached

* The dimensions of the indicator movements given were kindly furnished and revised by the manufacturers under whose names the proportions are written.

the links BD and AE; D is a stationary point on the frame of the instrument, and E is the articulation of AE with the end of the piston-rod G.

By taking the three links PB, BD, and AE of the proper proportions, a point S on AE will move in an arc of a circle when P is moved in a straight line parallel to the motion of E. Therefore,

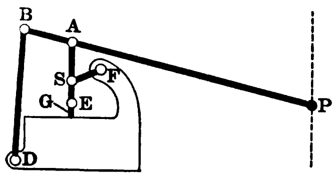


FIG. 134.

in order to construct the mechanism so that it will guide P in a straight path as indicated by the broken line, it is only necessary to add another link, SF, so that S will move in a circular arc about the stationary point F as a centre.

The proportions of the links must also be such that the velocities of P and E will have a constant ratio.

PROBLEMS.

1. Develop cycloidal tooth curves for a spur gear, pinion, and rack, the pinion being in mesh with both spur gear and rack.

Diametral pitch of spur and pinion	$1\frac{1}{2}$
Teeth in spur gear	40
" " pinion	16

Take rolling circle half as large as the pitch circle of a 15-tooth gear.

Lay out a tooth curve on each gear and the rack with the Robinson odontograph so as to compare with those found by using the describing circle.

Indicate path of contact and angle of action.

2. Develop teeth for a cycloidal annular gear and pinion, taking the number of teeth and rolling circle as in problem 1.

Indicate path of contact and angle of action.

3. Same as problem 1 except substituting involute for cycloidal teeth. Angle of obliquity = 15°.

4. Same as problem 2 except substituting involute for cycloidal teeth. Angle of obliquity = 15°.

5. Develop the teeth for a single-thread worm and worm-wheel.

Diameter of worm	4″
Pitch " "	$1\frac{1}{2}$″
Number of teeth on worm-wheel	42

6. Lay out the teeth for a double-thread worm and worm-wheel, cut templates and test for interference of teeth, finally making worm-wheel teeth of such a form that none of the flank is cut away.

Diameter of worm	= 4″
Pitch of worm	= 3″
Number of teeth in worm-wheel	= 12

7. Find the proper form of gear blanks and tooth curves for a pair of bevel gears running on shafts intersecting at right angles, having—

Velocity ratio........................ 3 : 4
Pitch diameter........................ 6″ and 8″
Diametral pitch...................... $2\frac{1}{2}$

Design cutters for both gears.

Draw two projections of gears, one in half-section on a plane parallel to both axes, the other in full projection on a plane perpendicular to one axis.

8. Design a pair of stepped pulleys for a crossed belt, having given—

Speed of driver...... 200 revolutions per minute.
" " driven..... 50, 100, 200, 400, 800 rev. per min.
Distance between centres..................... 30″
Diameter of smallest step..................... 4″

9. Plot diagram for determining cone-pulley diameters for an open belt.

10. Same as problem 8 except substituting an open belt for the crossed one.

Determine step diameters both by the diagram and the method of § 81.

Check results by calculating the length of belt.

11. Having given—

Distance between centres..................... 24″
The diameters 2″, 5″, 8″, 12″ of one cone and one diameter, 14″ of the other,

find the remaining steps of the second pulley.

Use both methods and check by calculating as in problem 10.

12. Design a cam to give the follower a reciprocating motion, uniformly accelerated upward during the first quarter of its revolution and uniformly retarded upward during the second quarter. The return motion to be the reverse of the upward. Axis of cam

and path of follower axis intersecting. Roller attached to follower to rest on face of cam. (See § 86.)

Stroke of follower..................... 4 inches.

13. Design a cam to give the follower a uniformly varied positive motion similar to that of problem 12. Roller on end of follower.

Stroke of follower............................ 6″
Distance between axes of cam and follower...... 1″

14. Design a cam to engage with the plane surface of a follower, the plane surface to move at right angles to itself with the same motion and length of stroke as in problem 12. (See § 89.)

15. Design a cam to operate a flat-face rocker-arm through an angle of 15°. Rocker-arm to remain stationary during one-third of the cam's revolution, and to rise and fall with uniformly varied motion during the remaining two-thirds of the revolution. (See § 89.)

Distance between cam and rocker arm shaft centre.. 10″

16. Design a cylindrical cam to give its follower the same motion as in problem 12.

Axes of cam and follower parallel.

17. Design a cylindrical cam to give a rocker-arm with attached roller the same motion as in problem 15.

Distance between axes........................ 10″

18. Design involute cam to give a mill-stamp 6″ drop.

Distance between cam- and stamp-shaft centres.. 5″

19. Whitworth quick-return shaper motion.

Maximum stroke............................ 8″
Length of connecting-rod..................... 16″
Time ratio for forward and return motion on maximum stroke..........................2 : 1
Centre of variably rotating crank $2\frac{1}{2}$″ above articulation of connecting rod and ram.

Construct velocity diagrams of ram for maximum and 4″ strokes. Scale of drawing = full size.

20. Crank and slotted-arm quick-return motion.

Maximum stroke............................ 18″
Length of connecting-rod.................... 12″
Time ratio for forward and return motion on maximum stroke......................... 3 : 2

Construct velocity diagram for maximum and 8″ strokes.

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